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ex 1 $\quad y' + 2y = e^{t} \qquad y(0) = 1$

$\boxed{\text{Method}}$

$$\mathcal{L}\{y' + 2y\} = \mathcal{L}\{e^{t}\}$$

$$\mathcal{L}\{y'\} + \mathcal{L}\{2y\} = \mathcal{L}\{e^{t}\}$$

$$\left[s\,\mathcal{L}\{y\} - y(0)\right] + 2\mathcal{L}\{y\} = \frac{1}{s-1}$$

Now $\quad \mathcal{L}\{y(t)\} = Y(s)$

$$\left[\underline{\underline{s\,Y(s)}} - 1\right] + 2\,\underline{\underline{Y(s)}} = \frac{1}{s-1}$$

$$(s+2)Y(s) - 1 = \frac{1}{s-1}$$

$$(s+2)Y(s) = \frac{1}{s-1} + 1 = \frac{1 + (s-1)}{s-1}$$

$$Y(s) = \frac{s}{(s-1)(s+2)} = \frac{1}{3}\,\frac{1}{s-1} + \frac{2}{3}\,\frac{1}{s+2}$$

$$\frac{-2}{-3} \quad \frac{\overset{1}{s}}{(s-1)(s+2)} = \frac{\overset{1/3}{A}}{s-1} + \frac{\overset{2/3}{B}}{s+2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \frac{1}{3}\,\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \frac{2}{3}\,\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$y(t) = \frac{1}{3}e^{t} + \frac{2}{3}e^{-2t}.$$

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$$y'' - 2y' = \boxed{-4}$$

$y(0)=0$

$y'(0)=0$

$$\left(s^2\underline{Y(s)} - \cancel{sy(0)} - \cancel{y'(0)}\right) - 2\left(s\underline{Y(s)} - \cancel{y(0)}\right) = -\frac{4}{s}$$

$$\boxed{\begin{aligned} \mathcal{L}\{y'\} &= sY(s) - y(0) \\ \mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) \end{aligned}}$$

$\mathcal{L}\{-4\} =$

$-4\mathcal{L}\{1\}$

$\frac{1}{s}$

$$(s^2 - 2s)Y(s) = -\frac{4}{s}$$

$$Y(s) = \frac{-4}{s^2(s-2)}$$

PFD $$\frac{-4}{s^2(s-2)} = \frac{\overset{1}{A}}{s} + \frac{\overset{2}{B}}{s^2} + \frac{\overset{-1}{C}}{s-2}$$

let $s=1$ $\quad 4 = A + 2 + 1$

$1 = A$

#16

t^n

$\frac{n!}{s^{n+1}}$

$$y(t) = 1 + 2t - e^{2t}.$$

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$$y'' - 6y' + 9y = te^{3t}$$

$y(0) = 3$

$y'(0) = 2$

$$\left(s^2 Y(s) - s\overset{3}{y(0)} - \overset{2}{y'(0)}\right) - 6\left(sY(s) - \overset{3}{y(0)}\right) + 9Y(s)$$

$$= \frac{1}{(s-3)^2}$$

$$\left(\underline{s^2 Y(s)} - 3s - 2\right) - 6\left(\underline{sY(s)} - 3\right) + 9\underline{Y(s)} = \frac{1}{(s-3)^2}$$

$$(s^2 - 6s + 9)Y(s) \underline{-3s + 16} = \frac{1}{(s-3)^2}$$

$$(s-3)^2 Y(s) = \frac{1}{(s-3)^2} + \underline{3s - 16}$$

$$(s-3)^2 Y(s) = \frac{1 + (3s-16)(s-3)^2}{(s-3)^2}$$

$$Y(s) = \frac{1 + (3s-16)(s-3)^2}{(s-3)^4}$$

$$\frac{1 + \overset{-7}{(3s-16)}(s-3)^2}{(s-3)^4} = \frac{\overset{-4}{A}}{s-3} + \frac{\overset{6}{B}}{(s-3)^2} + \frac{\overset{17}{C}}{(s-3)^3} + \frac{\overset{1}{D}}{(s-3)^4}$$

4

$$\left(\frac{2}{2}\right)\frac{1}{(s-3)^3}$$

let $s = 0$

$$\frac{-143}{81} = \frac{A}{-3} + \frac{B}{9} - \frac{C}{27} + \frac{1}{81}$$

$$\mathcal{L}^{-1}\left\{\frac{2}{(s-3)^3}\right\}$$

$$-143 = -27A + 9B - 3C + 1$$

$$\boxed{-144 = -27A + 9B - 3C}$$

$s = 2$

$s = 4$

$$y(t) = -4e^{3t} + 6te^{3t} + \frac{17}{2}\left[t^2 e^{+3t}\right]$$

$$+ \frac{1}{6} t^3 e^{3t}$$

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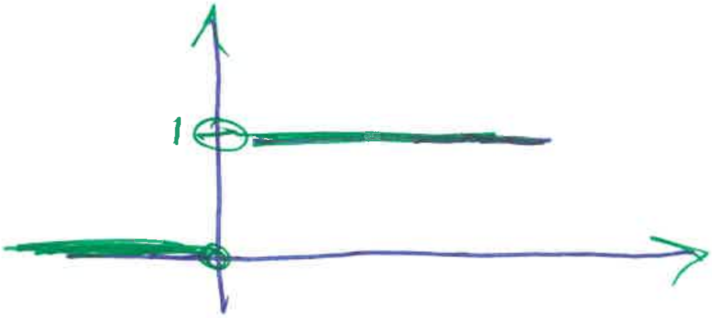
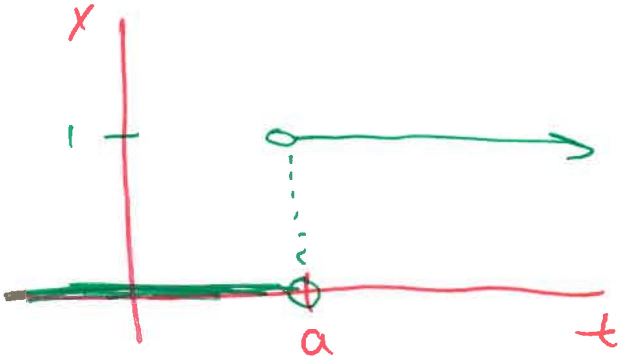
Unit Step function

$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$$



$$\star\ \mathcal{L}\{\underset{1}{u(t)}\} = \int_0^\infty e^{-st} u(t)\,dt = \int_0^\infty e^{-st}\,dt = \frac{1}{s}$$

$$u(t-a) = \begin{cases} 0, & t<a \\ 1, & t>a \end{cases}$$



$$\mathcal{L}\{u(t-a)\} = \int_0^\infty e^{-st}\,u(t-a)\,dt = \int_a^\infty e^{-st}\,dt$$

$$\lim_{N\to\infty}\int_a^N e^{-st}\,dt = -\frac{1}{s}\lim_{N\to\infty} e^{-st}\Big|_a^N =$$

$$-\frac{1}{s}\lim_{N\to\infty}\left[\cancel{\frac{1}{e^{sN}}} - e^{-as}\right]$$

$$\star\ \mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

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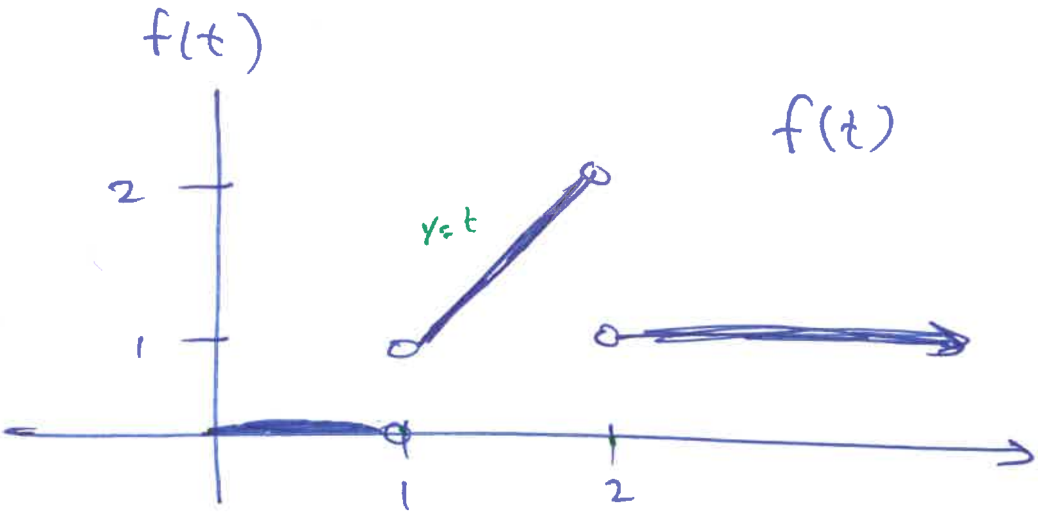

$$f(t) = \cancel{0\left[\overset{ON}{u(t)} - \overset{off}{u(t-1)}\right]} + t\left[\overset{ON}{u(t-1)} - \overset{off}{u(t-2)}\right]$$

$$+ 1\,u(t-2)$$

$$f(t) = t\,u(t-1) - t\,u(t-2) + u(t-2)$$

$$\boxed{f(t) = t\,u(t-1) - (t+1)\,u(t-2)}$$

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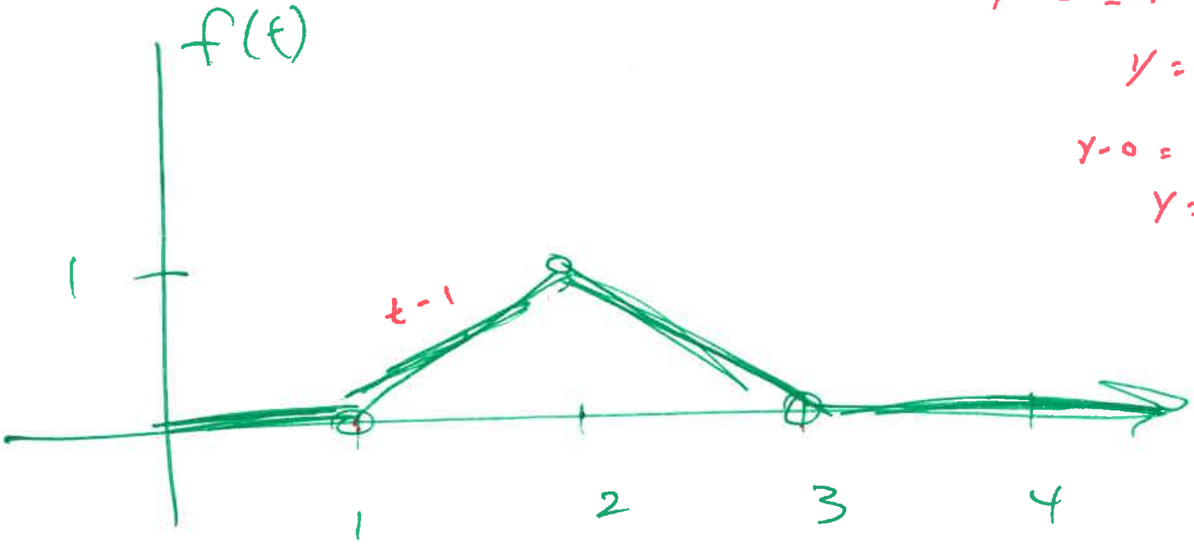
$y - 0 = 1(x-1)$

$y = x - 1$

$y - 0 = -(x-3)$

$y =$



$$f(t) = \cancel{0\,[u(t) - u(t-1)]} + (t-1)[u(t-1) - u(t-2)]$$

$$+ (-t+3)[u(t-2) - u(t-3)] + 0\,u(t-3)$$

$$(-t+1-t+3)\,u(t-2)$$

$$= \underline{(t-1)u(t-1)} - (t-1)\underline{u(t-2)} + (-t+3)\underline{u(t-2)}$$

$$- (-t+3)\,u(t-3)$$

$$= (t-1)u(t-1) + (-2t+4)\,u(t-2) + (t-3)u(t-3)$$

$$= \underline{(t-1)}\,\underline{u(t-1)} - 2\,\underline{(t-2)}\,\underline{u(t-2)} + \underline{(t-3)}\,u(t-3)$$

yes!

$$= e^{-s}\frac{1}{s^2} - 2e^{-2s}\frac{1}{s^2} + e^{-3s}\frac{1}{s^2}$$