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#30 State nature of Y_p for $y''-2y'+y=7e^{t}\cos t$

$y''-2y'+y=0$

Aux: $r^2-2r+1=0$

$(r-1)^2=0$

$r=1,\ r=1$

$\{e^{t},\ te^{t}\}$

$7e^{t}\underline{\cos t}$

$Y_p=(A\cos t+B\sin t)e^{t}$

$Y_p=A\underline{e^{t}\cos t}+B\underline{e^{t}\sin t}$

PLAY

$y''-2y'+y=5e^{t}+\underline{t^3}$

$\{\underline{e^{t}},\ te^{t}\}$

$Y_p=\underline{(At^3+Bt^2+Ct+D)}$

$+Et^2e^{t}$

$\cancel{(Et^2+Ft+G)e^{t}}$

$Y_p=At^3+Bt^2+Ct+D+Et^2e^{t}$

for this problem.

15.) $$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = te^t$$

AUX: $r^2 - 5r + 6 = 0$

UDC if $f(t)$ poly, exponential, sine or cosine or products of these

(1) $(r-2)(r-3) = 0$

$r = 2,\ r = 3$

$\{e^{2t}, e^{3t}\}$

☺

(2) * $y_p = (At + B)e^t$

$y_p' = (At+B)e^t + Ae^t = e^t(At + B + A)$

$y_p' = (At + (A+B))e^t$

$y_p'' = (At + (A+B))e^t + e^t(A)$

$y_p'' = (At + (2A+B))e^t$

$$(At + (2A+B))e^{t} - 5(At + (A+B))e^{t}$$

$$+ 6(At + B)e^{t} = te^{t}$$

$$2At + 2A + B - 5(A+B) + 6B = t$$

$$2At - 3A + 2B = t$$

$$A = \tfrac{1}{2} \qquad -3A + 2B = 0$$

$$-\tfrac{3}{2} + 2B = 0$$

$$B = \tfrac{3}{4}$$

$$y = C_1 e^{2t} + C_2 e^{3t} + \left(\tfrac{1}{2}t + \tfrac{3}{4}\right)e^{t}$$

4.6

#7 $\boxed{y''} + 4y' + 4y = \underbrace{e^{-2t}\ln t}_{g(t)}$

Aux: $r^2 + 4r + 4 = 0$

$$(r+2)^2 = 0$$

$r = -2, \quad r = -2$

$$\{\underbrace{e^{-2t}}_{y_1}, \underbrace{te^{-2t}}_{y_2}\}$$

$$W[e^{-2t}, te^{-2t}] = \begin{vmatrix} e^{-2t} & e^{-2t}t \\ -2e^{-2t} & e^{-2t} + (-2e^{-2t})t \end{vmatrix}$$

$$e^{-2t}(1-2t)$$

$$e^{-4t}(1-2t) + 2te^{-4t}$$

$$e^{-4t} - 2te^{-4t} + 2te^{-4t} = \boxed{e^{-4t}} \neq 0$$

$$V_1 = -\int \frac{y_2\, g(t)}{W(y_1, y_2)}\, dt = -\int \frac{(te^{-2t})(e^{-2t}\ln t)}{e^{-4t}}\, dt$$

$$= -\int t \ln t\, dt = -\left[\frac{1}{2}t^2 \ln t - \frac{1}{2}\int t\, dt\right]$$

IBP $\quad u = \ln t \qquad dV = t\, dt$

$\quad du = \frac{1}{t}\, dt \qquad V = \frac{1}{2}t^2$

$$= -\left[\frac{1}{2}t^2 \ln t - \frac{1}{4}t^2\right]$$

$$= -\frac{1}{2}t^2 \ln t + \frac{1}{4}t^2$$

$$\cancel{V_1 = \frac{1}{2}t} \qquad V_1 = \frac{1}{2}t^2\left(\frac{1}{2} - \ln t\right)$$

$$V_2 = \int \frac{y_1\, g}{W[y_1, y_2]}\,dt = \int \frac{(e^{-2t})(e^{-2t}\ln t)}{e^{-4t}}\,dt$$

$$V_2 = \int \ln t\,dt = t\ln t - t$$

$$= t(\ln t - 1)$$

$$y = \overbrace{c_1 e^{-2t} + c_2 t e^{-2t}}^{y_h} + \frac{1}{2}t^2\left(\frac{1}{2} - \ln t\right)e^{-2t} + t(\ln t - 1)\,t\,e^{-2t}.$$

CE

#12 $t^2 \frac{d^2 z}{dt^2} + 5t \frac{dz}{dt} + 4z = 0$

$y = t^n$

$a = 1 \qquad b = 5 \qquad c = 4$

AUX: $ar^2 + (b-a)r + c = 0$

$r^2 + 4r + 4 = 0$

$(r+2)^2 = 0$

$r = -2, \quad r = -2$

$\{t^{-2}, \; t^{-2} \ln t\}$

#14 $t^2 y'' - 3t y' + 4y = 0$

$a = 1 \qquad b = -3 \qquad c = 4$

Aux: $ar^2 + (b-a)r + c = 0$

$r^2 - 4r + 4 = 0$

$(r-2)^2 = 0$

$r = 2, \; r = 2$

$\{t^2, \; t^2 \ln t\}$

#11 $$\frac{d^2w}{dt^2} + \frac{6}{t}\frac{dw}{dt} + \frac{4}{t^2}w = 0$$

$$t^2\frac{d^2w}{dt^2} + 6t\frac{dw}{dt} + 4w = 0$$

$a = 1$ $b = 6$ $c = 4$

Aux: $r^2 + (6-1)r + 4 = 0$

$r^2 + 5r + 4 = 0$

$(r+4)(r+1) = 0$

$r = -4,\ r = -1$

$\{t^{-4}, t^{-1}\}$

#9 $t^2y'' + 2ty' - 6y = 0$

$a = 1$ $b = 2$ $c = -6$

Aux: $r^2 + (2-1)r - 6 = 0$

$r^2 + r - 6 = 0$

$(r+3)(r-2) = 0$

$r = -3,\ r = 2$

$\{t^{-3}, t^2\}$

CE

Aux: $r^2 + r + 7 = 0$

$t^2 y'' + 2t y' + 7y = 0$

$$r = \frac{-1 \pm \sqrt{1-28}}{2}$$

$$r = \frac{-1 \pm \sqrt{-27}}{2}$$

$$r = \frac{-1 \pm i\, 3\sqrt{3}}{2}$$

$$r = -\frac{1}{2} \pm i\, \frac{3\sqrt{3}}{2}$$

$$\left\{ t^{-1/2} \cos\left(\frac{3\sqrt{3}}{2} \ln t\right),\ t^{-1/2} \sin\left(\frac{3\sqrt{3}}{2} \ln t\right) \right\}$$

$$y = C_1 \qquad + C_2$$