#5

$$y'' + y = t^2 + 2 \qquad y(0) = 1 \;\; \& \;\; y'(0) = -1$$

$$\left(s^2 Y(s) - s + 1\right) + Y(s) = \frac{2}{s^3} + \frac{2}{s} = \frac{2s^2 + 2}{s^3}$$

$$Y(s)\left(s^2 + 1\right) - s + 1 = \frac{2s^2 + 2}{s^3}$$

$$Y(s)\left(s^2 + 1\right) = \frac{2s^2 + 2}{s^3} + (s - 1) = \frac{s^4 - s^3 + 2s^2 + 2}{s^3}$$

$$Y(s) = \frac{s^4 - s^3 + 2s^2 + 2}{s^3\,(s^2 + 1)}$$

So
$$\frac{s^4 - s^3 + 2s^2 + 2}{s^3\,(s^2 + 1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{\overset{2}{C}}{s^3} + \frac{Ds + E}{s^2 + 1}$$

then can find $C = 2$ by letting $s = 0$.

$$\frac{s^4 - s^3 + 2s^2 + 2}{s^3(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{2}{s^3} + \frac{Ds + E}{s^2+1}$$

Let $s = 1 \Rightarrow 2 = A + B + 2 + \frac{D+E}{2} \Rightarrow 2A + 2B + D + E = \phi$

Let $s = -1 \Rightarrow -3 = -A + B - 2 + \frac{-D+E}{2} \Rightarrow -2A + 2B - D + E = -2$

Let $s = 2 \Rightarrow \frac{9}{20} = \frac{A}{2} + \frac{B}{4} + \frac{1}{4} + \frac{2D+E}{5} \Rightarrow 10A + 5B + 8D + 4E = 4$

Let $s = -2 \Rightarrow -\frac{17}{20} = -\frac{A}{2} + \frac{B}{4} - \frac{1}{4} + \frac{-2D+E}{5} \Rightarrow -10A + 5B - 8D + 4E = -12$

we then have the system:

$$\left.\begin{array}{l} \left.\begin{array}{r} 2A + 2B + D + E = 0 \\ -2A + 2B - D + E = -2 \end{array}\right\} \; 2B + E = -1 \\ \left.\begin{array}{r} 10A + 5B + 8D + 4E = 4 \\ -10A + 5B - 8D + 4E = -12 \end{array}\right\} \; 5B + 4E = -4 \end{array}\right\} \quad \begin{array}{l} B = \phi \\ E = -1 \end{array}$$

Using equations 1 & 3 along with values for B, E

$$\left.\begin{array}{r} 2A + D = 1 \\ 5A + 4D = 4 \end{array}\right\} \quad \begin{array}{l} A = \phi \\ D = 1 \end{array}$$

So then $Y(s) = \frac{2}{s^3} + \frac{s-1}{s^2+1} = \frac{2}{s^3} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = \mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$y(t) = t^2 + \cos(t) - \sin(t).$$

check to make sure correct! (it is.)