

$$\#12 \quad w'' - 2w' + w = 6t - 2$$

$$w(-1) = 3$$

$$w'(-1) = 7$$

$$\text{Let } y(t) = w(t-1)$$

$$y'(t) = w'(t-1)$$

$$y''(t) = w''(t-1)$$

$$\underline{w''(t-1) - 2w'(t-1) + w(t-1) = 6(t-1) - 2}$$
$$= 6t - 8$$

So then

$$\boxed{y''(t) - 2y'(t) + y(t) = 6t - 8}$$

$$y(0) = 3$$

$$y'(0) = 7$$

$$\left(\int^2 Y(s) - 3s - 7 \right) - 2 \left(\int Y(s) - 3 \right) + Y(s) = \frac{6}{s^2} - \frac{8}{s} = \frac{6 - 8s}{s^2}$$

$$Y(s) (s^2 - 2s + 1) - 3s - 1 = \frac{6 - 8s}{s^2}$$

$$Y(s) (s^2 - 2s + 1) = \frac{6 - 8s}{s^2} + (3s - 1) = \frac{3s^3 - s^2 - 8s + 6}{s^2}$$

$$Y(s) = \frac{3s^3 - s^2 - 8s + 6}{s^2 (s-1)^2}$$

So now

$$\frac{3s^3 - s^2 - 8s + 6}{s^2(s-1)^2} = \frac{4}{s} + \frac{6}{s^2} + \frac{-1}{s-1} + \frac{0}{(s-1)^2}$$

$$\text{Let } s=2 \quad \left(\frac{10}{4} = \frac{A}{2} + \frac{6}{4} + C \right) 4$$

$$10 = 2A + 6 + 4C$$

$$4 = 2A + 4C$$

$$\underline{A + 2C = 2}$$

$$\text{Let } s=-1 \quad \left(\frac{10}{4} = -A + 6 - \frac{C}{2} \right) 4$$

$$10 = -4A + 24 - 2C$$

$$-14 = -4A - 2C$$

$$2A + C = 7$$

$$\begin{array}{l} A + 2C = 2 \\ 2A + C = 7 \end{array} \left. \vphantom{\begin{array}{l} A + 2C = 2 \\ 2A + C = 7 \end{array}} \right\}$$

$$-2A - 4C = -4$$

$$2A + C = 7$$

$$\hline -3C = 3$$

$$C = -1$$

$$A - 2 = 2$$

$$A = 4$$

$$\text{-then } \mathcal{L}^{-1} \left\{ \frac{3s^3 - s^2 - 8s + 6}{s^2(s-1)^2} \right\} = 4 \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + 6 \mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s-1} \right\}$$

$$y(t) = 4 + 6t - e^t$$

So we have

$$y(t) = 4 + 6t - e^t$$

and recall

$$y(t) = w(t-1) \text{ so } w(t) = y(t+1)$$

Then

$$y(t+1) = w(t) = 4 + 6(t+1) - e^{t+1}$$

$$w(t) = 10 + 6t - e^{t+1}$$
