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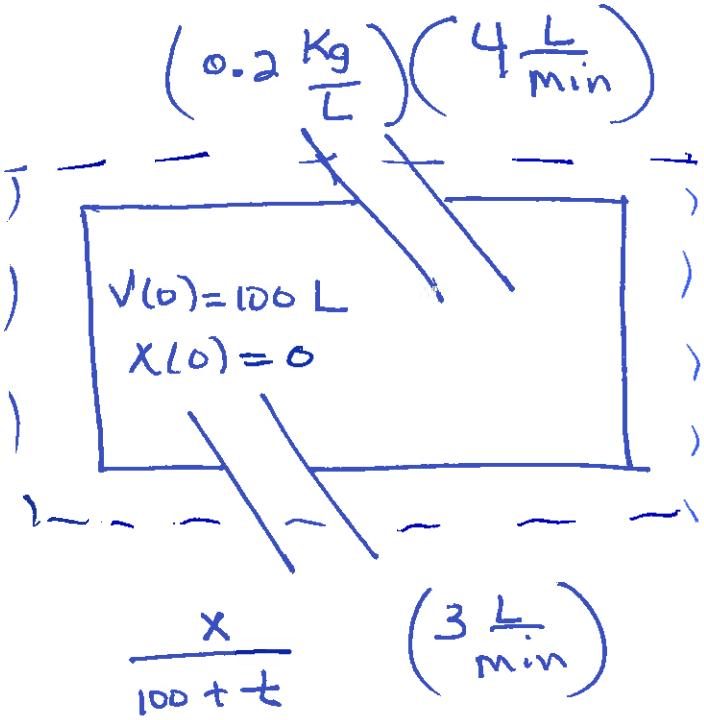
#4



Let

$X = x(t) =$ Kg of Salt in tank at time t

$$\frac{dx}{dt} = \dot{m}_{in} - \dot{m}_{OUT}$$

$$\frac{dx}{dt} = 0.8\,\frac{Kg}{min} - \frac{3x}{100+t}\,\frac{Kg}{mi}$$

Note this is really $100 + (1)t$

Net change in flow rate

$$\frac{dx}{dt} + \frac{3}{100+t}x = 0.8 \quad \therefore \text{ Linear}$$

here $P(t) = \dfrac{3}{100+t}$ & $3\displaystyle\int \frac{1}{100+t}\,dt = 3\ln(100+t)$

$= \ln(100+t)^3$

So $\mu(t) = e^{\int P(t)\,dt} = e^{\ln(100+t)^3} = (100+t)^3$

So then

$$(100+t)^3 \frac{dx}{dt} + \frac{3}{(100+t)^2} x = 0.8(100+t)^3$$

$$\frac{d}{dt}\left((100+t)^3 x\right) = 0.8(100+t)^3 \quad , \text{ integrate}$$

$$(100+t)^3 x = 0.2(100+t)^4 + C_1$$

$$x = 0.2(100+t) + \frac{C_1}{(100+t)^3}$$

when $t=0$, $x=0$ $\therefore$

$$0 = (0.2)(100) + \frac{C}{100^3}$$

$$0 = 20 + \frac{C}{100^3} \Rightarrow C = -20(100)^3$$

So $$\boxed{x = 0.2(100+t) - \frac{20(100)^3}{(100+t)^3}}$$

b) When will the concentration of salt in the tank reach $0.1\ \frac{Kg}{L}$?

we just found an expression for x, but x is Kg of salt at time t.

The concentration of salt at time t is the Kgs of salt divided by the volume of brine at time $t \Rightarrow \dfrac{x}{100+t}$

$$\therefore \quad \frac{x}{100+t} = \frac{0.2(100+t)}{100+t} - \frac{20(100)^3}{(100+t)^4}$$

& we want this to equal $0.1\ \frac{Kg}{L}$

So

$$0.1 = 0.2 - \frac{20(100)^3}{(100+t)^4}$$

$$-0.1 = -\frac{20(100)^3}{(100+t)^4}$$

$$-\frac{1}{10} = \frac{-20(100)^3}{(100+t)^4} \Rightarrow 200(100)^3 = (100+t)^4$$

$$100 + t = \left(200(100)^3\right)^{1/4}$$

$$t = \left(200(100)^3\right)^{1/4} - 100 \approx 18.9207115 \text{ min.}$$