#31 $y'' + y = t - (t-4)u(t-2)$, $\begin{cases} y(0) = 0 \\ y'(0) = 1 \end{cases}$

Take LT of both sides.

$$\left[s^2 Y(s) - \cancel{s\,y(0)} - \overset{1}{y'(0)}\right] + Y(s) = \frac{1}{s^2} - e^{-2s}\left(\frac{1}{s^2} - \frac{2}{s}\right)$$

where $\mathcal{L}\{(t-4)u(t-2)\} = e^{-2s}\mathcal{L}\{(t+2)-4\} = \underline{e^{-2s}\mathcal{L}\{t-2\}}$

$$Y(s)(s^2+1) - 1 = \frac{1}{s^2} - e^{-2s}\left(\frac{1-2s}{s^2}\right)$$

$$Y(s)(s^2+1) = \frac{1}{s^2} + 1 - e^{-2s}\left(\frac{1-2s}{s^2}\right)$$

$$Y(s)(s^2+1) = \frac{s^2+1}{s^2} - \left(\frac{1-2s}{s^2}\right)e^{-2s}$$

$$Y(s) = \frac{s^2+1}{s^2(s^2+1)} - \left(\frac{1-2s}{s^2(s^2+1)}\right)e^{-2s}$$

$$Y(s) = \frac{1}{s^2} + \frac{2s-1}{s^2(s^2+1)}e^{-2s}$$

Need to do a decomposition on e^{-2s} term.

#31

So we have

$$\frac{2s-1}{s^2(s^2+1)} = \frac{\overset{2}{A}}{s} + \frac{\overset{-1}{B}}{s^2} + \frac{\overset{-2}{C}s + \overset{1}{D}}{s^2+1}$$

we can find B quickly $\Rightarrow B = -1$

Let $s=1 \Rightarrow \frac{1}{2} = A - 1 + \frac{C+D}{2} \Rightarrow 2A + C + D = 3$

Let $s=-1 \Rightarrow -\frac{3}{2} = -A - 1 + \frac{-C+D}{2} \Rightarrow -2A - C + D = -1$

$2D = 2$

$\boxed{D=1}$

Let $s=2 \Rightarrow \frac{3}{20} = \frac{A}{2} - \frac{1}{4} + \frac{2C+1}{5} \Rightarrow 10A + 4C = 4$

$$\therefore \begin{cases} 2A + C = 2 \\ 10A + 4C = 4 \end{cases} \Rightarrow \begin{cases} -10A - 5C = -10 \\ 10A + 4C = 4 \end{cases}$$

$3C = -6$

$\boxed{C=-2}$

$C = -2 \Rightarrow \underline{A = 2}$

So we finally have

$$Y(s) = \frac{1}{s^2} + \left(\frac{2}{s} - \frac{1}{s^2} - 2\frac{s}{s^2+1} + \frac{1}{s^2+1}\right)e^{-2s}$$

Take $\mathcal{L}^{-1}$ of both sides

$$y(t) = t + \left[2 - (t-2) - 2\cos(t-2) + \sin(t-2)\right]u(t-2)$$

$$y(t) = t + \left[4 - t - 2\cos(t-2) + \sin(t-2)\right]u(t-2)$$