Math 231 10/14/2013

section 4.4

$y'' + 4y = 16t\sin(2t)$

(A) Consider $y'' + 4y = 0 \Rightarrow$ Aux: $r^2 + 4 = 0 \Rightarrow r = \pm 2i$, so a fund. Sol. Set is $\{\sin(2t), \cos(2t)\}$

B) The form of the particular solution is $y_p = t(At+B)\sin(2t) + t(Ct+D)\cos(2t)$

$y_p = (At^2 + Bt)\sin(2t) + (Ct^2 + Dt)\cos(2t)$, need y_p''.

$y_p' = 2(At^2+Bt)\cos(2t) + (2At + B)\sin(2t) - 2(Ct^2+Dt)\sin(2t) + (2Ct + D)\cos(2t)$

$y_p'' = -4(At^2+Bt)\sin(2t) + (4At + 2B)\cos(2t) + 2(2At + B)\cos(2t) + 2A\sin(2t)$

$\quad - 4(Ct^2+Dt)\cos(2t) - (4Ct + 2D)\sin(2t) - 2(2Ct + D)\sin(2t) + 2C\cos(2t)$

Now collect terms in y_p''.

$y_p'' = (-4At^2 + (-4B - 8C)t + 2A - 4D)\sin(2t) + (-4Ct^2 + (8A - 4D)t + 4B + 2C)\cos(2t)$

Now sub. into origional DE.

$$\left(-4At^2 + (-4B-8c)t + 2A-4D\right)\sin(2t) + \left(-4ct^2 + (8A-4D)t + 4B+2c\right)\cos(2t)$$

$$+ \left(4At^2+4Bt\right)\sin(2t) + \left(4ct^2+4Dt\right)\cos(2t) = 16t\sin(2t)$$

OR

$$\left(\cancel{-4At^2} + \cancel{4At^2} + \cancel{(-4B} - 8c + \cancel{4B)}t + 2A-4D\right)\sin(2t) +$$

$$\left(\cancel{-4ct^2} + \cancel{4ct^2} + (8A \cancel{-4D} \cancel{+4D})t + 4B+2c\right)\cos(2t) = 16t\sin(2t)$$

then $\left(-8ct + 2A-4D\right)\sin(2t) + \left(8At + 4B+2c\right)\cos(2t) = 16t\sin(2t)$

$\therefore$ $-8C=16$, $C=-2$ (1)

(2) $8A=0 \Rightarrow A=0$

(3) $4B+2C=0$

$4B-4=0$

$B=1$

(4) $2A-4D=0$

$D=0$

Since $A=0$ from (2)

$\therefore$ $A=0$, $B=1$, $C=-2$, $D=0$

$$y_p = t\sin(2t) - 2t^2\cos(2t)$$

$$y = y_h + y_p$$

$$\star\ y = C_1\sin(2t) + C_2\cos(2t) + t\sin(2t) - 2t^2\cos(2t)$$