Math 142 Numerical Handout -Trapezoidal Rule Spring 2003

Use the Trapezoidal rule to approximate $\int_1^2 \frac{1}{\sqrt{1+x^3}}\,dx$, with n = 6.

i	X_i	$f(x_i)$	M	M $f(x_i)$
0	1	0.70711	1	0.70711
1	7/6	0.62161	2	1.24323
2	8/6	0.54470	2	1.08941
3	9/6	0.47809	2	0.95618
4	10/6	0.42146	2	0.84293
5	11/6	0.37366	2	0.74733
6	2	0.33333	1	0.33333

Sum = 5.91952

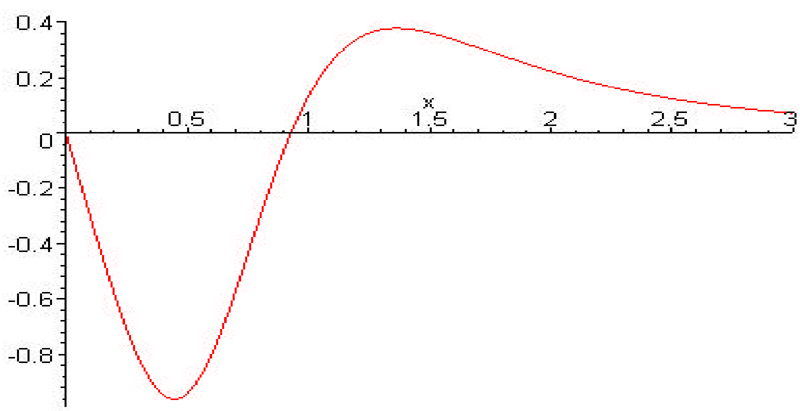
Then $\int_1^2 \frac{1}{\sqrt{1+x^3}}\,dx \cong \frac{5.91952}{12} = 0.49329$, now fnInt gives 0.49258.

We then have for the % error: $\frac{0.49329-0.49258}{0.49258} * 100 = 0.14\%$, not bad with n =6.

Now let's look at the error bound for the Trapezoidal rule for this example.

b-a = 1, n = 6 and here f"(x) = $\frac{27x^4}{(1+x^3)^{5/2}} - \frac{3x}{(1+x^3)^{3/2}}$ and from the graph below

we see that max $|f''(x)|$ over [1,2] is about 0.4.



So

$$|E_T| \le \frac{0.4*1}{12*36} = 0.00092596$$

Note in the above, if one uses the full display for the calculations the difference between fnInt and the trapezoidal approximation is 0.00071522992981 .