Math 142 Numerical Handout -Simpson's Rule Spring 2003

Use Simpson's rule to approximate $\int_1^2 \frac{1}{\sqrt{1+x^3}}\,dx$, with n = 6.

i	X_i	$f(x_i)$	M	$M\ f(x_i)$
0	1	0.70711	1	0.70711
1	7/6	0.62161	4	2.48646
2	8/6	0.54470	2	1.08941
3	9/6	0.47809	4	1.91237
4	10/6	0.42146	2	0.84293
5	11/6	0.37366	4	1.49466
6	2	0.33333	1	0.33333

Sum = 8.86626

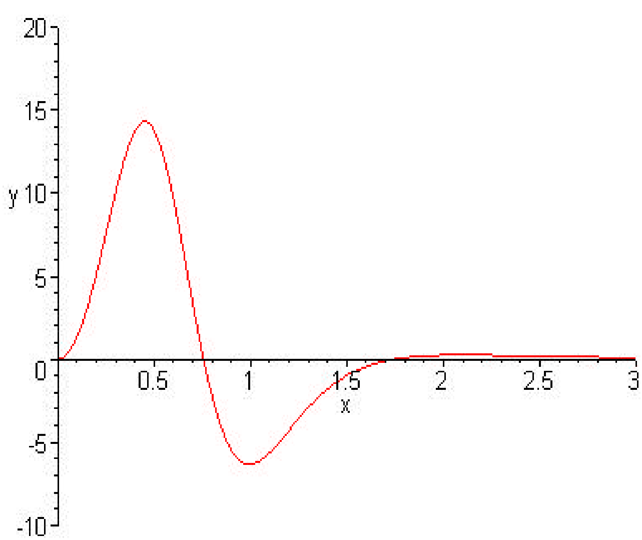
Then $\int_1^2 \frac{1}{\sqrt{1+x^3}}\,dx \cong \frac{8.86626}{18} = 0.49257$, now fnInt gives 0.49258.

Now let's look at the error bound using Simpson's rule for this example.

b-a = 1, n = 6 and here $f^4(x) = \frac{8505\,x^8}{16(1+x^3)^{9/2}} - \frac{1215\,x^5}{2(1+x^3)^{7/2}} + \frac{135\,x^2}{(1+x^3)^{5/2}}$ and from the graph below we see that max $\left|f^4(x)\right|$ over [1,2] is about 8.

So then we have for the max error:

$\left|E_S\right| \le \frac{8}{180*6^4} = 0.0000342936$.



Again, if we keep the full display all through the above table and with fnInt, the difference is seen to be only 0.00000801162852.