

- 1) Determine whether this sequence is convergent or divergent. If convergent, determine the limit.

$$\{(-2)^n\}_{n=1}^{\infty}$$

This is a sequence with $r = -2 < -1$, so this sequence diverges.

- 2) Use the integral test to determine the convergence or divergence of:

$$\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^2 - 1}}$$

$$\int_2^{\infty} \frac{x}{\sqrt{x^2 - 1}} dx = \lim_{A \rightarrow \infty} \int_2^A \frac{x}{\sqrt{x^2 - 1}} dx = \lim_{A \rightarrow \infty} \left[\sqrt{x^2 - 1} \right]_2^A = \lim_{A \rightarrow \infty} \left[\sqrt{A^2 - 1} - \sqrt{1} \right] = \infty$$

The integral diverges, so then the series diverges.

3) Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{2^n}{n^3 + 1}$

Ratio Test gives 2
So divergent.

b) $\sum_{n=1}^{\infty} \frac{e^{2n}}{(2n-1)!}$

Ratio Test gives 0
So convergent.

c) $\sum_{n=1}^{\infty} 3^{1/n}$

Divergent by Test for
Divergence.

d) $\sum_{n=1}^{\infty} \frac{2^{-n-1}}{5^{-n}}$

Geometric, $r = 5/2 > 1$,
Divergent.

- 4) Determine whether this series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1}$$

- i) First test for absolute convergence: (Ratio Test gives 1, NO HELP)

Consider, $\sum \frac{\sqrt{n}}{n+1}$, divergent by Limit Comparison Test with $\sum \frac{1}{\sqrt{n}}$. So this series is not absolutely convergent.

- ii) Next consider conditional convergence. The series is alternating, with the terms decreasing and the limit is zero as $n \rightarrow \infty$, so by the Alternating Series Test the series is conditionally convergent.

5) Determine the radius and interval of convergence for:

$$\sum_{n=1}^{\infty} \frac{\ln n}{e^n} (x-e)^n$$

By the Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{(\ln n + 1)(x-e)^{n+1}}{e^{n+1}} \cdot \frac{e^n}{(\ln n)(x-e)^n} \right| = \frac{|x-e|}{e} \lim_{n \rightarrow \infty} \left| \frac{\ln(n+1)}{\ln n} \right| < 1 \quad \text{for convergence.}$$

So $\frac{|x-e|}{e} < 1 \Rightarrow |x-e| < e$, so the radius of convergence is “e”.

When $x = 2e$, we have: $\sum \ln n$, which is divergent (Test for Divergence).

When $x = 0$, we have: $\sum (-1)^n \ln n$, which is divergent (Test for Divergence).

So the interval of convergence is $I = (0, 2e)$.

