Exam 3 Solutions Math 142 Spring 03

1) Let $f(x) = \frac{x^2}{\ln(x+1)}$, then consider $\lim_{x\to\infty} \frac{x^2}{\ln(x+1)}$

$\lim_{x\to\infty} \frac{x^2}{\ln(x+1)} \overset{LR}{=} \lim_{x\to\infty} \frac{2x}{1/(x+1)} = \lim_{x\to\infty} 2x(x+1) = \infty$, so the sequence diverges to infinity.

2) $\int_2^\infty \frac{1}{x(\ln x)^2}\,dx = \lim_{A\to\infty} \int_2^A \frac{1}{x(\ln x)^2}\,dx = -\lim_{A\to\infty} \frac{1}{\ln x}\bigg|_2^A = -\lim_{A\to\infty}\left[\frac{1}{\ln A} - \frac{1}{\ln 2}\right] = \frac{1}{\ln 2}$

let u = ln x
du = 1/x dx giving $\int u^{-2}\,du$

so since the integral is convergent, then so is the series convergent.

3) (a) here r = -3/2, the series is geometric with |r| > 1 and divergent.

(b) note, $0 \le \frac{1-\cos n}{n^2} \le \frac{2}{n^2}$ and the series $2\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series so the given series is convergent by the comparison test.

(c) Convergent by Alternating Series Test since:

i) $\lim_{x\to\infty} \frac{\sqrt{\ln x}}{x} \overset{LR}{=} \lim_{x\to\infty} \frac{1}{2x\sqrt{\ln x}} = 0$, so $\lim_{n\to\infty} \frac{\sqrt{\ln n}}{n} = 0$.

ii) If we let $f(x) = \frac{\sqrt{\ln x}}{x} \Rightarrow f'(x) = \frac{1-2\ln x}{x^2\sqrt{\ln x}} < 0$ when 1-2ln x is less than zero, or when $x > \sqrt{e}$. So f(x) is decreasing for x > 3.

4) Using the ratio test:

$$\lim_{n\to\infty}\left|\frac{9^{n+1}(x-2)^{n+1}}{n+2}\bullet\frac{n+1}{9^n(x-2)^n}\right| = 9|x-2|\lim_{n\to\infty}\left|\frac{n+1}{n+2}\right| = 9|x-2|,$$ then for absolute convergence we must have $9|x-2|<1$. Now this means we have absolute convergence for any x in the interval $-\frac{1}{9}<x-2<\frac{1}{9} \quad\Rightarrow\quad \frac{17}{9}<x<\frac{19}{9}$.

Lets check the endpoints:

If x = 17/9, then the series becomes: $\sum_{n=0}^{\infty}(-1)^n\frac{1}{n+1}$ which converges since this is the Alternating Harmonic Series.

If x = 19/9, then the series becomes: $\sum_{n=0}^{\infty}\frac{1}{n+1}$ which diverges since this is just the Harmonic series.

So the interval of convergence is $\left[\frac{17}{9},\frac{19}{9}\right)$.

5) Let f(x) = 10^x, find $T_4(x)$.

$T_4(x)$ = $\sum_{n=0}^{4}\frac{f^{n}(0)}{n!}x^n = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{iv}(0)}{4!}x^4$.

$f(x) = 10^x$ $\quad$ $f(0) = 1$

$f'(x) = 10^x \ln 10$ $\quad$ $f'(0) = \ln 10$

$f''(x) = 10^x(\ln 10)^2$ $\quad$ $f''(0) = (\ln 10)^2$

$f'''(x) = 10^x(\ln 10)^3$ $\quad$ $f'''(0) = (\ln 10)^3$

$f^{iv}(x) = 10^x(\ln 10)^4$ $\quad$ $f^{iv}(0) = (\ln 10)^4$

so $T_4(x) = 1 + \frac{\ln 10}{1!}x + \frac{(\ln 10)^2}{2!}x^2 + \frac{(\ln 10)^3}{3!}x^3 + \frac{(\ln 10)^4}{4!}x^4$

$T_4(x) = 1 + 2.30x + 2.65x^2 + 2.03x^3 + 1.17x^4$.