Math 142 Exam 4 Sp08 Name

No Work-No Credit 9am Last 4 Digits

1) Determine whether this sequence is convergent or divergent. If convergent evaluate the limit.

$$a_n = \frac{e^n}{2^{n-1}}$$

We need to find $\lim_{n\to\infty} \frac{e^n}{2^{n-1}} = 2\lim_{n\to\infty} \frac{e^n}{2^n} = 2\lim_{n\to\infty}\left(\frac{e}{2}\right)^n = \infty$, since $\frac{e}{2} > 1$.

So divergent.

2a) Complete the writing of the following series in sigma notation.

$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

b) Use the Integral test to determine the convergence or divergence of this series.

We have

$$\int_1^{\infty} \frac{1}{(2x-1)^2}\,dx = \lim_{A\to\infty}\int_1^{A} \frac{1}{(2x-1)^2}\,dx = -\frac{1}{2}\lim_{A\to\infty} \frac{1}{2x-1}\Big|_1^A = -\frac{1}{2}\lim_{A\to\infty}\left[\frac{1}{2A-1} - 1\right] = \frac{1}{2}$$

So convergent.

3) Determine whether this series is (i) absolutely convergent, (ii) conditionally convergent or (iii) divergent.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$$

i) First test for absolute convergence. The Ratio Test fails. We must consider $\sum |a_n| \Rightarrow \sum_{n=1}^{\infty}\frac{n}{n^3+1}$. Using the Limit Comparison Test with $\sum\frac{1}{n^2}$ we have $\lim_{n\to\infty}\frac{\frac{n}{n^3+1}}{\frac{1}{n^2}}=\lim_{n\to\infty}\frac{n^3}{n^3+1}=1$, this series is convergent, and the original series is absolutely convergent.

4) Determine if these series are convergent or divergent. (No work-No credit)

a) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{1.3n}$ = $\frac{1}{1.3}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$, which is just the alternating harmonic series and thus convergent.

b) $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots$ = $\sum_{n=0}^{\infty}\frac{1}{4^n}=\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$, geometric, r = ¼ , so convergent.

c) $\sum_{n=1}^{\infty}\frac{1+2n}{n(n+1)}$, Use the LCT with $\sum\frac{1}{n} \Rightarrow \lim_{n\to\infty}\frac{\frac{1+2n}{n^2+n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n+2n^2}{n^2+n}=2$

so the original series is divergent.

5) For $\sum_{n=0}^{\infty}(-1)^n \frac{1}{3^n \sqrt{n+1}}(x-4)^n$, determine:

a) Radius of convergence, R.

$\lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{3^{n+1}\sqrt{n+2}} \cdot \frac{3^n \sqrt{n+1}}{(x-4)^n}\right| = \frac{|x-4|}{3}\lim_{n\to\infty}\left|\frac{\sqrt{n+1}}{\sqrt{n+2}}\right| = \frac{|x-4|}{3}$, and for absolute convergence we need $\frac{|x-4|}{3} < 1 \Rightarrow |x-4| < 3$, so R = 3.

b) Interval of convergence, I.

We now have absolute convergence for x $\in$ (1, 7). We need to check the endpoints.

Let x = 1 $\Rightarrow \sum_{n=0}^{\infty}(-1)^n \frac{(-3)^n}{3^n \sqrt{n+1}} = \sum_{n=0}^{\infty}\frac{3^n}{3^n \sqrt{n+1}} = \sum_{n=0}^{\infty}\frac{1}{\sqrt{n+1}}$, now use LCT with $\sum \frac{1}{\sqrt{n}}$, a divergent p-series, and we see that this series is divergent.

Let x = 7 $\Rightarrow \sum_{n=1}^{\infty}(-1)^n \frac{3^n}{3^n \sqrt{n+1}} = \sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}}$, which is convergent by the AST.

So I = (1, 7].