Math 142 Exam 2 Sp08 Name

No Work-No Credit (9AM) Last 4 Digits

1) Use the Trapezoidal Rule to approximate $\int_0^{\pi/2} \sqrt{1+\sin x}\, dx$, with n = 3.

(Must complete the table for ***any*** credit.)(Do not round your values!)

i	x_i	$f(x_i)$	M	$Mf(x_i)$
0	0	1	1	1
1	$\frac{\pi}{6}$	1.224744871	2	2.449489743
2	$\frac{2\pi}{6}$	1.366025404	2	2.732050808
3	$\frac{\pi}{2}$	1.414213562	1	1.414213562

$\Delta x = \frac{\pi}{6}$

$\sum M_i f(x_i) = 7.595754113.$

By the Trapezoidal Rule $\int_0^{\pi/2} \sqrt{1+\sin x}\, dx \approx \frac{\pi}{12}(7.595754113) = 1.988563777$

2) Determine the partial fraction decomposition of $\frac{5x+11}{(x+1)(x^2+1)}$.

$\frac{5x+11}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+1}$, so A = 3, B = - 3, C = 8.

3) Evaluate: $\int (x^2+2)\ln x \, dx$.

$$\int (x^2+2)\ln x \, dx = \left(\frac{x^3}{3}+2x\right)\ln x - \int\left(\frac{1}{3}x^2+2\right)dx = \left(\frac{x^3}{3}+2x\right)\ln x - \frac{x^3}{9} - 2x + C.$$

Let u = ln x   dv = (x^2+2) dx

du = $\frac{1}{x}$dx   v = $\frac{x^3}{3}+2x$

4) Determine the value of the integral or show it is divergent:

$$\int_0^3 \frac{x}{(x^2-4)^{2/3}}dx = \lim_{A\to 2^-}\int_0^A \frac{x}{(x^2-4)^{2/3}}dx + \lim_{B\to 2^+}\int_B^3 \frac{x}{(x^2-4)^{2/3}}dx$$

Next evaluate the integral: $\int \frac{x}{(x^2-4)^{2/3}}dx = \frac{1}{2}\int u^{-2/3}du = \frac{3}{2}u^{1/3} = \frac{3}{2}(x^2-4)^{1/3}$

Let u = x^2 -4, then du = 2x dx so ½ du = x dx

Then we have

$$\frac{3}{2}\lim_{A\to 2^-}(x^2-4)^{1/3}\Big|_0^A + \frac{3}{2}\lim_{B\to 2^+}(x^2-4)^{1/3}\Big|_B^3 =$$

$$\frac{3}{2}\lim_{A\to 2^-}\left\{(A^2-4)^{1/3}-(-4)^{1/3}\right\} + \frac{3}{2}\lim_{B\to 2^+}\left\{(5)^{1/3}-(B^2-4)^{1/3}\right\} =$$

$\frac{3}{2}\left(4^{1/3}+5^{1/3}\right)$, so convergent.

5) Evaluate: $\int \frac{\sqrt{x^2-4}}{x}dx = 2\int \frac{2\tan\theta}{2\sec\theta}2\sec\theta\tan\theta\, d\theta = 2\int \tan^2\theta\, d\theta$ =

$$2\int(\sec^2\theta - 1)d\theta = 2(\tan\theta+\theta)+C \Rightarrow 2\left(\frac{\sqrt{x^2-4}}{2}+\sec^{-1}\left(\frac{x}{2}\right)\right)+C.$$

Let x = 2sec θ, dx = 2sec θ tan θ