Math 142 Exam 2 F06 Name

No Work-No Credit Last 4 digits

1) Evaluate: $\int x^5 (\ln x)\, dx$.

$$\int x^5 (\ln x) dx = \frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$$

Let u = ln x   dv = x^5 dx

$$du = \frac{1}{x} dx \qquad v = \frac{x^6}{6}$$

2) Find the partial fraction decomposition for $\frac{3x^2 + x - 2}{x^3 - 4x}$.

(No Work-No Credit)

$$\frac{3x^2 + x - 2}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2} \quad \Rightarrow \quad A = \tfrac{1}{2},\ B = 1,\ C = \frac{3}{2}.$$

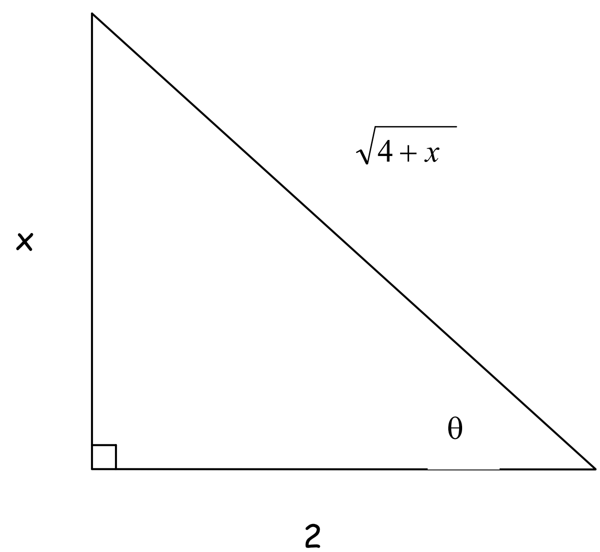
3) Evaluate: $\int \frac{1}{\left(4+x^2\right)^{\frac{3}{2}}}\,dx$

$$\int \frac{1}{\left(4+x^2\right)^{\frac{3}{2}}}\,dx = \frac{1}{4}\int \frac{1}{\left(1+\tan^2\theta\right)^{3/2}}\sec^2\theta\,d\theta = \frac{1}{4}\int \frac{1}{\sec^3\theta}\sec^2\theta\,d\theta = \frac{1}{4}\int \cos\theta\,d\theta =$$

$$\frac{1}{4}\sin\theta + C = \frac{1}{4}\frac{x}{\sqrt{4+x^2}} + C.$$

Let x = 2tan θ

dx = $2\sec^2\theta\,d\theta$



4) Compute the value of the integral, or prove it divergent.

$$\int_0^{\frac{\pi}{2}} \sec^2 x \, dx = \lim_{A \to \frac{\pi}{2}^-} \int_0^A \sec^2 x \, dx = \lim_{A \to \frac{\pi}{2}^-} \tan x \Big|_0^A = \lim_{A \to \frac{\pi}{2}^-} \left(\tan(A) - \tan(0)\right) =$$

$= \lim_{A \to \frac{\pi}{2}^-} \tan(A) = \infty$, so divergent.

5) Use Simpson's Rule to approximate $\int_1^2 \sin\left(e^x\right) dx$, let n = 4.

(Must complete the table for credit.)

i	x_i	$f(x_i)$	M	$Mf(x_i)$
0	1	0.410781290503	1	0.410781290503
1	1.25	-0.34172360956	4	-0.34172360956
2	1.5	-0.973506592772	2	-0.973506592772
3	1.75	-0.504309917978	4	-0.504309917978
4	2	0.893854954913	1	0.893854954913

$\sum Mf(x_i) =$ -4.02651105028

$$\int_1^2 \sin\left(e^x\right) dx \approx \left(\frac{0.25}{3}\right)(-4.02651105\ \ 028) = -0.3355425875\ \ 24$$