Math 142 Exam 1 Sp07 Name

No Work-No Credit 11:15 Last 4 Digits

1) On the moon, the acceleration due to gravity is a(t) = - 1.6 $\frac{m}{s^2}$. A golf ball is dropped from a height of 115.2 m on the moon and falls to the surface.

a) Determine an equation giving the height of the golf ball at time, t.

v(t) = -1.6t + C_1, since v(0) = 0 $\Rightarrow$ C_1 = 0 and v(t) = -1.6t.

h(t) = -0.8t^2 + C_2, and we know that h(0) = 115.2 $\Rightarrow$ C_2 = 115.2, this gives

h(t) = -0.8t^2 + 115.2.

b) When will it hit the surface of the moon?

When the golf ball hits the surface of the moon, h = 0 and 0 = -0.8t^2 + 115.2

and t = 12s.

2) Determine: $\frac{d}{dx}\int_{\frac{x}{e^x}}^{x^4} \sin^2(t)\,dt$.

$$\frac{d}{dx}\int_{\frac{x}{e^x}}^{x^4} \sin^2(t)\,dt = \left(\sin^2\left(x^4\right)\right)\left[4x^3\right] - \left(\sin^2\left(\frac{x}{e^x}\right)\right)\left[\frac{e^x - xe^x}{\left(e^x\right)^2}\right].$$

3) Use the data in this table to approximate $\int_0^{10} J(x)\,dx$.

x	0	3	6	9	12	15
J(x)	30	22	12	-4	-20	-38

a) Evaluate using the left endpoints of each subinterval: $\sum_{i=0}^{4} J(x_i)\Delta x$.

$\sum_{i=0}^{5} J(x_i)\Delta x$ = (30 + 22 + 12 – 4 – 20)(3) = 120.

b) Evaluate using the right endpoints of each subinterval: $\sum_{i=1}^{5} J(x_i)\Delta x$.

$\sum_{i=1}^{5} J(x_i)\Delta x$ = (22 + 12 – 4 – 20 – 38)(3) = - 84.

4) Show that $\int x^2 e^x \; dx = e^x\left(x^2 - 2x + 2\right) + C$ is correct.

$$\frac{d}{dx}\left(e^x\left(x^2 - 2x + 2\right)\right) = e^x\left[2x - 2\right] + \left(x^2 - 2x + 2\right)\left[e^x\right] = x^2 e^x.$$

5) Evaluate, for definite integrals, answers must be exact:

a) $\int -5x\sqrt{3-x^2}\,dx = -5\int x\sqrt{3-x^2}\,dx = \frac{5}{2}\int u^{1/2}du = \frac{5}{3}u^{3/2} + C = \frac{5}{3}\left(3-x^2\right)^{3/2} + C$.

Let u = 3 – x^2 $\Rightarrow$ du = -2x dx $\Rightarrow$ $-\frac{1}{2}du = x\,dx$.

b) $\int_{-1}^{2}\left(1+x-x^3\right)dx$ = $\left((2)+\frac{(2)^2}{2}-\frac{(2)^4}{4}\right)-\left((-1)+\frac{(-1)^2}{2}-\frac{(-1)^4}{4}\right)=\frac{3}{4}$.

c) $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$ = $\int \frac{1}{\sqrt{1-u^2}}du = \sin^{-1}(u) + C = \sin^{-1}\left(e^x\right) + C$.

Let u = e^x $\Rightarrow$ du = e^x dx.