

Let x = speed of the train in miles per hour.

a) We are given the cost per mile is proportional to the square of the speed:

$\$ = kx^2$ and when $x = 24$, the cost is \$18 $\Rightarrow k = \frac{1}{32}$. So then we will have

$c(x) = \frac{1}{32}x^2 + 72$ the units here being $\frac{\$}{\text{mile}}$. To get the correct units we need

$$\frac{c(x)}{x} = \frac{1}{32}x + \frac{72}{x} = \frac{1}{32}x + 72x^{-1} = D(x).$$

$$\text{Then } D'(x) = \frac{1}{32} - 72x^{-2} = \frac{1}{32} - \frac{72}{x^2} = 0 \quad \Rightarrow x = 48 \text{ mph.}$$

Now note that $D''(x) = 144x^{-3}$ and $D''(48) > 0$ so we have a minimum.