1) Given \( x^2 - x - 1 = \frac{1}{x+1} \), use the Intermediate Value Theorem to show there is a solution for \( 1 < x < 2 \).

Let \( f(x) = x^2 - x - 1 - \frac{1}{x+1} \). Then \( f(1) = -1.5 \) and \( f(2) = 0.6 \), so there must be a “c” in the interval \((1, 2)\) such that \( f(c) = 0 \).

2) Complete this definition:

a) \( f(x) \) is continuous at a number \( x = a \) if \( \lim_{x \to a} f(x) = f(a) \).

b) Let \( f(x) = \begin{cases} 
2, & x \leq -1 \\
ax + b, & -1 < x < 3 \\
-2, & x \geq 3
\end{cases} \). Find all values of \( c \) such that \( f \) is continuous on \( \mathbb{R} \).

Note that for all \( x \) except \(-1\) & 3 \( f(x) \) is continuous, so investigate there.

We need \( \lim_{x \to -1} f(x) = \lim_{x \to -1} (2) = 2 = -a + b = \lim_{x \to -1} (ax + b) \).

And this gives (i) \( 2 = -a + b \), also we need
\( \lim_{x \to -1} f(x) = \lim_{x \to -1} (ax + b) = 3a + b = -2 = \lim_{x \to -3} (ax + b) \) giving (ii) \( 3a + b = -2 \).

So now we need to solve (i) and (ii) \( \begin{cases} 
-a + b = 2 \\
3a + b = -2
\end{cases} \) in the first equation, solve for \( b \), \( b = a + 2 \) then

Substitute this into the second equation, \( 3a + (a + 2) = -2 \) giving \( a = -1 \) and back substituting gives \( b = 1 \).
3) Evaluate, if possible: 
\[ \lim_{x \to 2} \frac{x^4 - 16}{x - 2} . \]

\[ \lim_{x \to 2} \frac{x^4 - 16}{x - 2} = \lim_{x \to 2} \frac{(x^2 + 4)(x - 2)(x + 2)}{x - 2} = \lim_{x \to 2} (x^2 + 4)(x + 2) = 32 \]

4a) State the formal \( \varepsilon, \delta \) definition of the limit, \( \lim_{x \to a} f(x) = L \).

\( \lim_{x \to a} f(x) = L \) means that for every \( \varepsilon > 0 \) there is a \( \delta > 0 \) such that if \( 0 < |x - a| < \delta \) then \( |f(x) - L| < \varepsilon \).

b) Use part (a) to determine: 
\[ \lim_{x \to 3} \left( \frac{x - 1}{2} \right) = -2 . \]

Proof: Let \( \varepsilon > 0 \). Choose \( \delta = 2\varepsilon \). Then whenever \( 0 < |x - 3| < \delta \) we will have \[ \left| \left( \frac{x - 1}{2} - 2 \right) \right| < \varepsilon . \]

5) Given \( f(x) = \frac{2}{x} \), find \( f'(x) \) using \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \).

\[ \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{2}{x + h} - \frac{2}{x} = \lim_{h \to 0} \frac{2x - 2x - 2h}{h(x + h)} = \lim_{h \to 0} \frac{-2}{x(x + h)} = \frac{-2}{x^2} \]
6) **Given** \( f(x)=\sqrt{x^3+1} \) \& \( f'(x) = \frac{3x^2}{2\sqrt{x^3+1}} \), find the linear approximation, \( L(x) \), at \( x = 2 \). That is, find \( L(x) \), such that \( L(x) \approx f(x) \) as long as \( x \) is near 2.

\( f(2) = \sqrt{8+1} = 3 \), so the point on \( f \) is \((2,3)\) and the slope of the line tangent is \( f'(2) = 2 \). So the equation of the line tangent is \( y - 3 = 2(x-2) \Rightarrow y = 2x - 1 \).

So \( L(x) = 2x - 1 \). Then if \( x \) is very near 2, then \( \sqrt{x^3+1} \approx 2x-1 \).

7) **Evaluate, if possible:**

\[
\lim_{x \to \infty} 4000 \frac{x^3 + 500x^2}{x^4+1} =
\]

\[
\lim_{x \to \infty} 4000 \left( \frac{x^3}{x^4} + \frac{500}{x^4} \right) = 4000 \lim_{x \to \infty} \left( \frac{1}{x} + \frac{500}{x^4} \right) = 0.
\]

8) **For** \( k(x)=\frac{3x^2+4x+5}{x^2+8x-20} \), determine:

a) **Vertical asymptotes (if any):**

\[
k(x)=\frac{3x^2+4x+5}{x^2+8x-20} = \frac{3x^2+4x}{x^2+8x} = \frac{3x^2+4x}{(x+10)(x-2)}
\]

so \( x = -10 \) \& \( x = 2 \) are vertical asymptotes.

b) **Horizontal asymptotes (if any):**

\[
\lim_{x \to \infty} \frac{3x^2+4x+5}{x^2+8x} = \lim_{x \to \infty} \frac{3 + \frac{4}{x} + \frac{5}{x^2}}{1 + \frac{8}{x} - \frac{20}{x^2}} = 3.
\]

So \( y = 3 \) is a horizontal asymptote.