

1) Given the parametric equations, eliminate the parameter  $t$ .

$$x = \sec(t) \quad y = \tan(t), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

$$\text{Now } \tan^2(t) + 1 = \sec^2(t) \Rightarrow y^2 + 1 = x^2.$$

2) Complete the formal definition of the derivative of a function:

$$\text{a) } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

b) Use (a) to find the derivative of  $f(x) = x^2 + 16x$ .

$$f'(x) = \lim_{h \rightarrow 0} \frac{((x+h)^2 + 16(x+h)) - (x^2 + 16x)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{(x^2 + 2xh + h^2 + 16x + 16h) - (x^2 + 16x)}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 16h}{h} = \lim_{h \rightarrow 0} \frac{h(2x + h + 16)}{h} =$$

$$\lim_{h \rightarrow 0} (2x + h + 16) = 2x + 16.$$

3) (a) Complete the (**formal**) definition:  $\lim_{x \rightarrow a} f(x) = L$  means for every  $\epsilon > 0$ , there exists a  $\delta > 0$  such that if  $0 < |x - a| < \delta$  then  $|f(x) - L| < \epsilon$ .

b) Use (a) to show  $\lim_{x \rightarrow 2} (7 - 2x) = 3$ .

Pf:

Let  $\epsilon > 0$ . Choose  $\delta = \frac{\epsilon}{2}$ . Then if

$0 < |x - 2| < \delta$  we will have  $|(7 - 2x) - 3| < \epsilon$ .

Show work here.

We want  $|(7 - 2x) - 3| < \epsilon$ . Then

$$|4 - 2x| = |-2(x - 2)| = |-2||x - 2| = 2|x - 2|$$

And then  $2|x - 2| < \epsilon$  or  $|x - 2| < \frac{\epsilon}{2}$ .

4) Evaluate the limit, if possible.

$$\text{a) } \lim_{x \rightarrow \infty} \frac{4 - \frac{1}{2}x}{x + 4} = \lim_{x \rightarrow \infty} \frac{4 - \frac{1}{2}x}{x + 4} \cdot \left( \frac{\frac{1}{x}}{\frac{1}{x}} \right) = \lim_{x \rightarrow \infty} \frac{\frac{4}{x} - \frac{1}{2}}{1 + \frac{4}{x}} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}.$$

$$\text{b) } \lim_{x \rightarrow 4^+} \frac{3 - 2x}{x - 4} = \lim_{x \rightarrow 4^+} (3 - 2x) \frac{1}{x - 4} = -\infty.$$

5) Given the graph of  $f(x)$  below evaluate:

$$\text{a) } \lim_{x \rightarrow -4^+} f(x) = 4$$

$$\text{b) } \lim_{x \rightarrow -4^-} f(x) = -2$$

$$\text{c) } f(-4) = 2$$

$$\text{d) } \lim_{x \rightarrow 0^+} f(x) = -3$$

$$\text{e) } \lim_{x \rightarrow 0^-} f(x) = 2$$

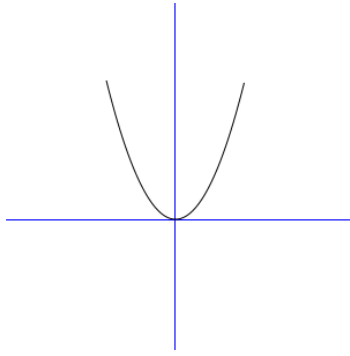
$$\text{f) } f(0) = 4$$

6) Use the Intermediate Value Theorem to show there is a root of the given equation in the specified interval.

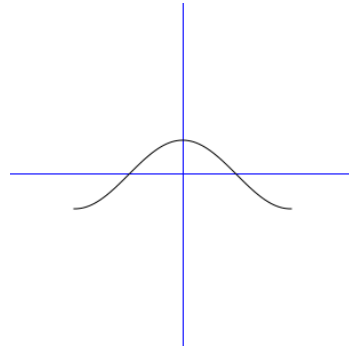
$x^3 = e^x$ , for  $x \in (1, 2)$ . Let  $f(x) = x^3 - e^x$ . Now  $f(1) < 0$ , and  $f(2) > 0$ . Since  $f(x)$  is continuous for all  $x \in (1, 2)$  by the IVT there is a  $c \in (1, 2)$  such that  $f(c) = 0$ .

7) On the left are graphs of functions and on the right are the graphs of the derivatives of these functions. Match the graph of the function to its derivative.

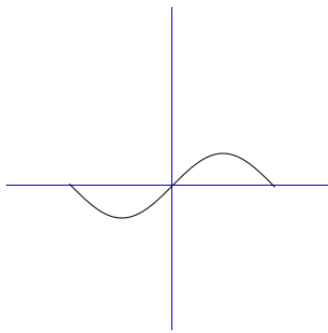
a)



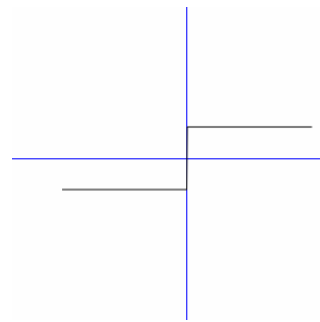
(i)



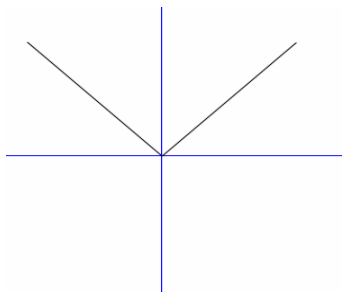
b)



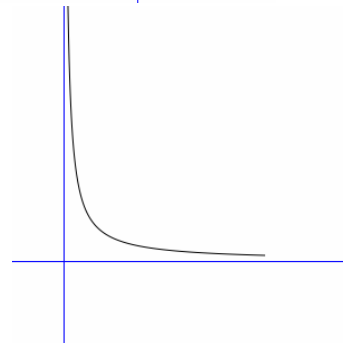
ii)



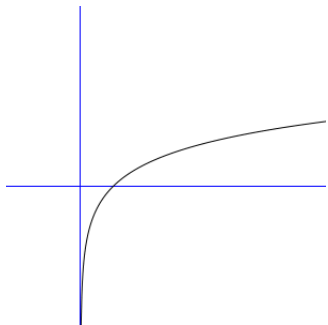
c)



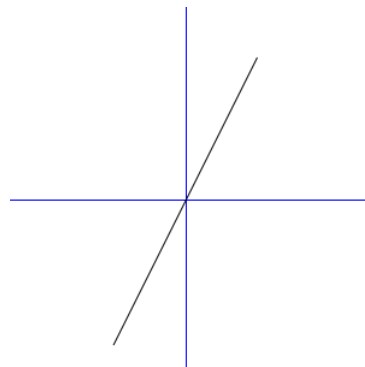
(iii)



d)



(iv)



a) iv

b) i

c) ii

d) iii