#11, 5.2:

$$\begin{array}{r} -2 \\ (D^2-1) \end{array} \begin{cases} (D^2-1)[u] + 5v = e^t \\ 2u + (D^2+2)[v] = 0 \end{cases}$$

$$\Rightarrow \quad -10v + (D^2-1)(D^2+2)[v] = (e^t)(-2)$$

$$\Rightarrow \quad (D^4 + D^2 - 12)[v] = -2e^t$$

<u>homogeneous part v_h:</u>

try $v_h = e^{rt} \Rightarrow r^4 + r^2 - 12 = 0 \Rightarrow (r^2+4)(r^2-3) = 0$

$$\Rightarrow \boxed{r = \pm 2i, \pm\sqrt{3}}$$

$\Rightarrow$ real valued solutions are: $v_h = c_1 \cos(2t) + c_2 \sin(2t) + c_3 e^{\sqrt{3}t} + c_4 e^{-\sqrt{3}t}$

<u>nonhomog. part v_p:</u>

try $v_p = Ae^t \Rightarrow A + A - 12A = -2 \Rightarrow -10A = -2 \Rightarrow A = 1/5$

so $\boxed{v = c_1 \cos(2t) + c_2 \sin(2t) + c_3 e^{\sqrt{3}t} + c_4 e^{-\sqrt{3}t} + \frac{1}{5}e^t}$

Now solving the 2nd ODE for u gives:

$$u = -\frac{1}{2}(D^2+2)[v] = -\frac{1}{2}D^2 v - v$$

so $u = -\frac{1}{2}\left[-4c_1\cos(2t) - 4c_2\sin(2t) + 3c_3 e^{\sqrt{3}t} + 3c_4 e^{-\sqrt{3}t} + \frac{1}{5}e^t\right]$

$\quad - \left[c_1\cos(2t) + c_2\sin(2t) + c_3 e^{\sqrt{3}t} + c_4 e^{-\sqrt{3}t} + \frac{1}{5}e^t\right]$

$$\Rightarrow \boxed{u = c_1\cos(2t) + c_2\sin(2t) - \frac{5}{2}c_3 e^{\sqrt{3}t} - \frac{5}{2}c_4 e^{-\sqrt{3}t} - \frac{3}{10}e^t}$$

<u>ASIDE:</u>

Notice that if I let $\boxed{u = d_1\cos(2t) + d_2\sin(2t) + d_3 e^{\sqrt{3}t} + d_4 e^{-\sqrt{3}t} - \frac{3}{10}e^t}$

so that $d_1 = c_1$, $d_2 = c_2$, $d_3 = -\frac{5}{2}c_3$ $d_4 = -\frac{5}{2}c_4$, then

$$\boxed{v = d_1\cos(2t) + d_2\sin(2t) - \frac{2}{5}d_3 e^{\sqrt{3}t} - \frac{2}{5}d_4 e^{-\sqrt{3}t} + \frac{1}{5}t}$$

* this is the form in which the solution is given at the back of the book

#15, section 5.4

$$\begin{cases} \dfrac{dx}{dt} = 2x + y + 3 & \text{(A)} \\ \dfrac{dy}{dt} = -3x - 2y - 4 & \text{(B)} \end{cases}$$

(I) Nullclines: $2x+y+3=0 \Rightarrow y=-2x-3$ (A)

$-3x-2y-4=0 \Rightarrow y = -\tfrac{3}{2}x-2$ (B)

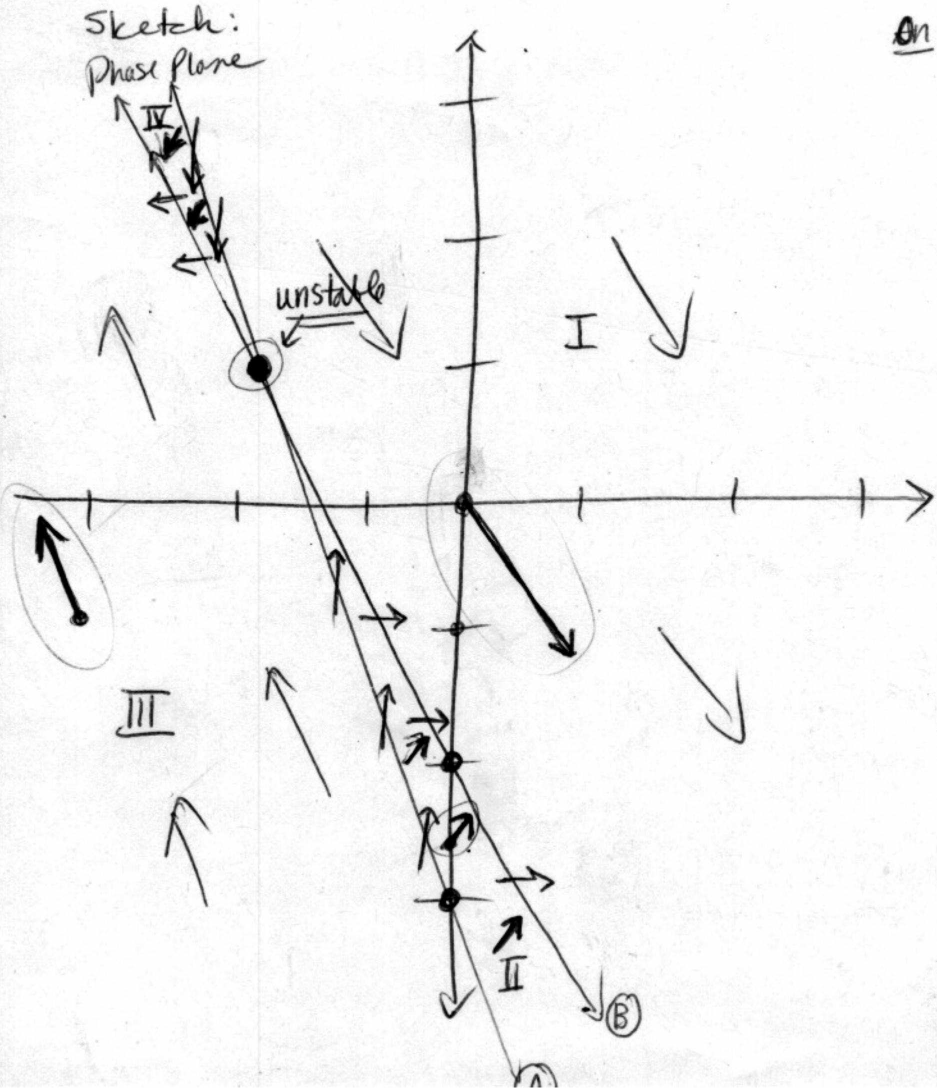
Equilibria $\Rightarrow$ both $\frac{dx}{dt}=0$ and $\frac{dy}{dt}=0 \Rightarrow y = \boxed{-2x-3 = -\tfrac{3}{2}x-2}$

$\Rightarrow \tfrac{1}{2}x = -1 \Rightarrow x=-2$

$\Rightarrow y = 1$

$\Rightarrow$ equilibrium pt @ $(-2,1)$

Sketch:
Phase plane



On the (A) nullcline, $\frac{dx}{dt}=0$, and $y=-2x-3$

$\Rightarrow \frac{dy}{dt} = -3x-2(-2x-3)-4$

$= x+2$

so if $\begin{cases} x>-2 \Rightarrow \frac{dy}{dt}>0 \\ x<-2 \Rightarrow \frac{dy}{dt}<0 \end{cases}$

On the (B) nullcline, $\frac{dy}{dt}=0$ and $y=-\tfrac{3}{2}x-2$

$\Rightarrow \frac{dx}{dt} = 2x+(-\tfrac{3}{2}x-2)+3$

$= \tfrac{1}{2}x+1$

so $\begin{cases} \frac{dx}{dt}>0 \text{ if } x>-2 \\ \frac{dx}{dt}<0 \text{ if } x<-2 \end{cases}$

region I:
@ $(0,0) \Rightarrow \begin{cases} \frac{dx}{dt}=3>0 \\ \frac{dy}{dt}=-4<0 \end{cases}$

region III:
$(-3,-1)$
$\frac{dx}{dt}=-4<0$
$\frac{dy}{dt}=7>0$

region II: @ $(0,-2.5)$
$\begin{cases} \frac{dx}{dt}=0.5>0 \\ \frac{dy}{dt}=0.5>0 \end{cases}$

region IV:
@ $(-4,4.5)$
$\frac{dx}{dt}=-0.$ [illegible]
$\frac{dy}{dt}=$ [illegible]

#17, section 5.4

$$\begin{cases} \dfrac{dx}{dt} = 2x + 13y & \text{(A)} \\ \dfrac{dy}{dt} = -x - 2y & \text{(B)} \end{cases} \Rightarrow$$

Nullclines: $y = -\frac{2}{13}x$ (A)

$y = -\frac{1}{2}x$ (B)

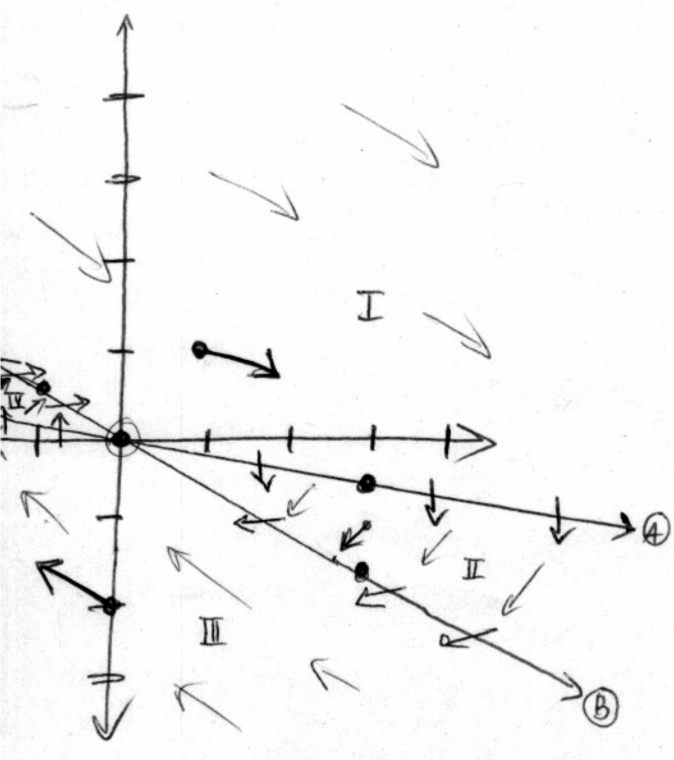
equilibria:

$-\frac{2}{13}x = -\frac{1}{2}x$

$\Rightarrow x = 0$

$\Rightarrow y = 0$

☆ $(0,0)$ is only equilib. pt.



on nullcline (A), $\frac{dx}{dt} = 0$ and $y = -\frac{2}{13}x$

$\Rightarrow \frac{dy}{dt} = -x - 2(-\frac{2}{13}x) = -\frac{9}{13}x$

$\Rightarrow \begin{cases} \frac{dy}{dt} < 0 \text{ if } x > 0 \\ \frac{dy}{dt} > 0 \text{ if } x < 0 \end{cases}$

on nullcline (B), $\frac{dy}{dt} = 0$ and $y = -\frac{1}{2}x$

$\Rightarrow \frac{dx}{dt} = 2x + 13(-\frac{1}{2}x) = -\frac{9}{2}x$

$\Rightarrow \begin{cases} \frac{dx}{dt} < 0 \text{ if } x > 0 \\ \frac{dx}{dt} > 0 \text{ if } x < 0 \end{cases}$

in region I: @ $(1,1) \Rightarrow \begin{cases} \frac{dx}{dt} = 15 > 0 \\ \frac{dy}{dt} = -3 < 0 \end{cases}$

in region II: @ $(3,-1) \Rightarrow \begin{cases} \frac{dx}{dt} = -7 < 0 \\ \frac{dy}{dt} = -1 < 0 \end{cases}$

in region III: @ $(0,-2) \Rightarrow \begin{cases} \frac{dx}{dt} = -26 < 0 \\ \frac{dy}{dt} = 4 > 0 \end{cases}$

in region IV: @ $(-2, \frac{1}{2})$

$\Rightarrow \begin{cases} \frac{dx}{dt} = 2.5 > 0 \\ \frac{dy}{dt} = 1 > 0 \end{cases}$

guess that equilib. @ $(0,0)$ is stable.