Find the first three nonzero terms in the Taylor poly. approx to the sol'n of

$$\begin{cases} y' = \sin(y) + e^x \\ y(0) = 0. \end{cases}$$

for x near zero.

Solution: we want $y(x) \approx y(0) + y'(0)x + y''(0)\frac{x^2}{2} + \cdots + \frac{y^{(N)}(0)}{N!}x^N$.

for some N so that the poly has three nonzero coeffs.

We know $y(0) = 0$. Since $y' = \sin(y) + e^x$ for all x,

we have $y'(0) = \sin(y(0)) + e^0 = \sin(0) + e^0 = 1$

and since $y'' = \cos(y)y' + e^x$

$$\Rightarrow y''(0) = \cos(y(0)) \cdot y'(0) + e^0$$
$$= \cos(0) \cdot 1 + e^0 = 2.$$

Now, $y''' = -\sin(y)(y')^2 + \cos(y) \cdot y'' + e^x$

so $y'''(0) = -\sin(y(0)) \cdot (y'(0))^2 + \cos(y(0)) \cdot y''(0) + e^0$

$$= -\sin(0) \cdot 1^2 + 1 \cdot 2 + 1 = 3$$

Since we have 3 nonzero coeffs now, we can stop and

$$\boxed{y(x) \approx x + \frac{2}{2!}x^2 + \frac{3}{3!}x^3}$$ is our approximate solution for x near 0.