# MATH 606 - Homework #2– due April 22, 2005

(1) Consider the PDE $u_t = u_{xx}$ for $x \in [0, 1]$ with boundary conditions $u(0, t) = u(1, t) = 0$ for all $t$. Let the initial condition be $u(x, 0) = sin(\pi x)$ so that the general solution is $u(x, t) = e^{\pi^2 t} sin(\pi x)$.

   (a) Write a program to solve this PDE for $t \in [0, 0.5]$, using the forward-time, central-space scheme. Compare the numerical solution at $t = 0.5$ with the analytical solution. Do this for three different values of $r = k/h^2$, namely $r = 0.1, 0.25, and 0.5$. Generate the table for values of $u_{exact}(0.5, 0.5) - u_{approx}(0.5, 0.5)$:

   (b) Plot the numerical solution for $h = 0.1$ and $r = 1$ at $t = 0.5$.

   (c) Now let your initial condition be the same as in the book for example 2.1. Plot the numerical solution for $h = 0.1$ and $r = 1$ at $t = 0.05$ and 0.10.

(2) Modify the code from (1) to solve $u_t = au_{xx} + g(x, t)$ for $x \in [0, 1]$ with boundary conditions $u(0, t) = u_L$ and $u(1, t) = u_R$. Here $g(x, t)$ is some function which you will code by using a function file, so that you can change $g$ without changing the rest of the code. Test your code by letting $a = 2$ and the exact solution be $u(x, t) = 5 - 2x + x(1 - x)e^{-t}$. Choose $g$ to satisfy $g(x, t) = u_t - au_{xx}$. Again, compare the exact and approximate solutions at $t = 0.5$ and $x = 0.5$ for the same values of $h$ as in (1).

(3) Solve the same inital-value problem as in (1), this time using Crank-Nicolson. It is your responsibility to find, or code, a function to solve the resulting tridiagonal matrix system.

(a) Compare the exact solution and numerical solution at $t = 0.5$ for very different choices of $r$. For one fixed value of $r$, choose some initial $h$ and repeatedly halve it to show that the error is really $O(h^2)$.

(b) Compare the exact solution and numerical solution at $t = 0.5$ for very different choices of $\rho = k/h$. For one fixed value of $\rho$, choose some initial $h$ and repeatedly halve it to show that the error is really $O(h^2)$.

(4) Consider the equation $u_t = u_{xx}$ for $x \in [0, 1]$ and for all $t \geq 0$ with mixed boundary conditions $a_L u_x(0, t) + b_L u(0, t) = c_L$ and $a_R u_x(0, t) + b_R u(0, t) = c_R$ . Write down the forward-time central-space explicit difference equation (by hand) as the matrix equation

$$\vec{u}_{j+1} = A\vec{u}_j + \vec{d}_j$$

where $\vec{u}_j = (u_{0,j}, u_{1,j}, ....., u_{n,j})'$ in two ways:

(a) discretize $u_x(0, t)$ and $u_x(1, t)$ using second-order centered differences

(b) discretize $u_x(0, t)$ and $u_x(1, t)$ using second-order forward difference and backward difference respectively. (this part will be much messier!)

Please zip all m-files into one file called "hw2" and email to me at heather@math.ohio-state.edu by 11:00 am Friday, April 22. Handwritten files should be turned in by the beginning of class the same day.