Solutions:

Taubes Problems –

1. If I know the graph of $u(1,x) = y$, then I can approximate some values for u and various values of x, say for two values of x, $x = x_1$ and x_2. Suppose that I get that $u(1,x_1) \approx y_1$ and $u(1,x_2) \approx y_2$. Then use the fact that for all x we have the formula

   $$u(1,x) = Re^{-x^2/(4\mu)}$$

   so that in particular

   $$y_1 \approx Re^{-x_1^2/(4\mu)}$$

   and

   $$y_2 \approx Re^{-x_2^2/(4\mu)}$$

   We now have a system of two equations with two unknowns (R and μ) that we can solve. Notice that possibly the easiest way to do this is to take $x_1 = 0$ if we can. Then we get that $y_1 \approx R$, and we only need work to solve for μ.

2. Case $c > 0$: Here $B(x) = c_1 e^{\sqrt{c}x} + c_2 e^{-\sqrt{c}x}$. Thus $B'(x) = c_1\sqrt{c}e^{\sqrt{c}x} - c_2\sqrt{c}e^{-\sqrt{c}x}$ and $B''(x) = c_1 c e^{\sqrt{c}x} + c_2 c e^{-\sqrt{c}x} = cB$. Thus the ODE $B'' = cB$ is satisfied by this choice of B.

   Case $c = 0$: Here $B(x) = ax + b$, so $B'(x) = a$ and $B''(x) = 0 = cB$. Again the ODE $B'' = cB$ is satisfied by this choice of B.

   Case $c < 0$: Here $B(x) = c_1\cos(\sqrt{-c}x) + c_2\sin(\sqrt{-c}x)$. Thus $B'(x) = -c_1\sqrt{-c}\sin(\sqrt{-c}x) + c_2\sqrt{-c}\cos(\sqrt{-c}x)$, and $B''(x) = c_1 c\cos(\sqrt{-c}x) + c_2 c\sin(\sqrt{-c}x) = cB$. Again, the ODE $B''(x) = cB(x)$ is satisfied by our choice of B.

4. (a) If $B(x) = \alpha e^{5x} + \beta e^{-5x}$, then $B(0) = \alpha + \beta = 0$ and $B(1) = \alpha e^5 - \alpha e^{-5} = 0$, which implies that $\alpha(e^5 - e^{-5}) = 0$ so that $\alpha = 0$ must be true. Thus $B(x) = 0$.

   (c) If $B(x) = \alpha\cos(\pi x) + \beta\sin(\pi x)$, then $B(0) = \alpha = 0$ and $B(1) = -\alpha = 0$. So $\alpha = 0$ must be true and β can be any real number.

   (d) If $B(x) = \alpha\cos(2\pi x) + \beta\sin(2\pi x)$, then $B(0) = \alpha = 0$ and $B(1) = \alpha = 0$ and again β is free to be any real number.

Problem 12 from Parkhurst Chapter 11:

$$u_t = Mu_{xx} + (b - f)u$$

Here every term in the equation ends up having units of (pop. density)/day if u represents the population density of the insects at position x and time t.