

Labwork Set # 6 – Math 371 – Fall 2009 Due date: 11/ 23 /2009

1. [14 points] Write a MATLAB function `quads.m` that will take in the endpoints of an interval $[a, b]$ and a case number and returns a vector of the approximations of the integral of a function f (f is specified by the case number) over $[a, b]$ computed via each of the following rules:

- (i) Midpoint Rule
- (ii) Trapezoid Rule
- (iii) Simpson's Rule
- (iv) Extrapolated Simpson's Rule

The function options should be

- (i) $\int_0^1 3x + 2 dx$
- (ii) $\int_0^1 x^2 + 3x + 2 dx$
- (iii) $\int_0^1 x^3 dx$
- (iv) $\int_0^1 x^4 dx$
- (v) $\int_0^1 e^x dx$.

For the all of the integrals, compute the true solution by hand, and compare the results of the quadratures to the true value. Where it's relevant explain how the order of the method allows you to predict some of the outcomes. [suggestion: you can use something like “(true integral)*ones(4)- (vector of approxs)” to get all the errors in one step..also feel free to write a function file that does it all at once]

2. [7 points] As you might remember from Calculus, e^{-x^2} is not an integrable function (meaning I can't write down an explicit formula for it's antiderivates). It's integral, however, shows up all over the place in real world problems. The MATLAB function “`erf`” is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-p^2} dp .$$

- (Step 1) Write a new function using the function from number (1) that takes in a value of x and a number of subdivisions N , and returns a vector of approximations for $erf(x)$, computed compositely over subintervals of length $\frac{x-0}{N}$.
- (Step 2) Modify the function from Step 1 so that you can also easily see the error in each of the approximations.
- (Step 3) Again modify it to find by what factor the error decreases if N changes from N to $2N$ for each quadrature rule. Does this correspond with the theoretical order of the method? Fill in the table, letting $x = .4$ and N change between $N = 1, 2, 4, 8$:

<i>N</i> change/ <i>Method</i>	<i>Midpoint</i>	<i>Trapezoid</i>	<i>Simpson's</i>	<i>E.Simpson</i>
1 → 2				
2 → 4				
4 → 8				

[NOTE: you need only turn in the final version of the function, not one version for each step. When run, it should return the vector of approximations, show the error for each approximation, and show the ratio of the errors for N and $2N$.

3. [points] In this problem, we will find and analyze approximate solutions of

$$y' + y = \sin(2t) .$$

Note, the true solution is $y(t) = -\frac{2}{5} \cos(2t) + \frac{1}{5} \sin(2t) + Ce^{-t}$, where C is determined by the initial condition. Let $y(0) = 0$.

- (a) Find C .
 - (b) Write a MATLAB m-file that computes the approximate solution of this ODE over the interval $[0,10]$ via Euler's method with step size h , computes the actual error between the true and approximate solutions at time $t_1 = h$ and at time $t = 10$, and plots the approximate and true solutions together on one graph (with one labeled so it's clear which is which). You can determine what the structure of the function file should be (what it's inputs, outputs should be, etc).
 - (c) Run your file from part (b) with step sizes $h = 2, 1, .5, .25, .125, .0625$ and verify that the order of the local truncation error is h^2 and the global truncation error is approximately order h as $h \rightarrow 0$. Are there issues with stability?
 - (d) Repeat parts (b) and (c) for Backward Euler's method.
4. Write a MATLAB m-file to compute the approximate solution for problem 2 in the written homework. If the true solution is $y(t) = -t - 1 + e^t$, verify the order of the error, and plot the true versus approximate solutions for at least 3 values of h .
5. Use the implicit trapezoid method

$$y_{n+1} = y_n + \frac{h}{2}(f(t_n, y_n) + f(t_{n+1}, y_{n+1})) ,$$

to solve the problem $y' = \lambda y$, with $y(0) = y_0$ as in homework problem number 3. Illustrate the stability ranges you found by taking appropriate values of h and λ to show that certain choices do/do not yield stable methods.