Labwork Set \# 5 - Math 371 - Fall 2009 Due date: Friday 11/6/2009

1. [4 points] Do problem 5.6 from Moler's book, parts (a) and (b) only. You can use MATLAB predefined functions to check the rank.
2. [4 points] Suppose you have the matrix problem $X \vec{\beta}=\vec{y}$ and you want to solve for $\beta$ in the least squares sense. Write a MATLAB function that takes in an arbitrary $m \times n$ matrix $X$, with $m>n$, and an arbitrary vector $\vec{y}$ and returns a least square solution. You should obtain your solutions via Householder reflections (do not use any predefined MATLAB function in order to do this, at least in the full rank case). Make sure that you handle rank deficient matrices appropriately. Test your code with at least two examples where you know what the solution should be to see if it is functioning properly. Also test it with a rank deficient matrix.
3. [4 points] A planet approximately follows an elliptical orbit: $x^{2}+b x y+c y^{2}+d x+e y+f=0$. Here are ten observations of it's position in the $(x, y)$ plane:

| $x$ | 1.02 | .95 | .87 | .77 | .67 | .56 | .44 | .30 | .16 | .01 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 0.39 | .32 | .27 | .22 | .18 | .15 | .13 | .12 | .13 | .15 |

(a) Determine the coefficients of the ellipse for this planet's orbit by fitting the ellipse to the data in the least squares sense. Plot the obtained ellipse in the $(x, y)$ plane along with the data points.
(b) This least squares problem is nearly rank deficient. To see what effect this has on the solution, perturb the data slightly by adding to each coordinate of each data point a random number selected from $[-.0005, .0005]$. Compute the new coefficients resulting from the perturbed data and plot the new orbit along with the old orbit.
4. [4 points] Problem 5.7 from Moler's book.

