Labwork Set # 1 – Math 371 – Fall 2009 Due date: 9/8/2009

1. Write the following ".m" function files:

   (a) [5 points] A function that takes in an arbitrary vector and a norm type (either 1,2,or inf), and returns the norm of the vector (using the type of norm specified in the input).

   (b) [5 points] A function that takes in an arbitrary matrix and a norm type (again, either 1,2,or inf), and returns the norm of the matrix.

   (c) [3 points] A function that takes in an arbitrary **square, invertible** matrix and a norm type, and uses part (b) to return the condition number of the matrix. Take one well-conditioned matrix and one ill-conditioned matrix and compare the values you get from your condition number function with the built-in value from the MATLAB function cond(A,p).

   Note: do not use built-in MATLAB functions that calculate the norms or condition number for you!

2. Note: this problem is connected to problems 2 and 3 of the written homework. Define the matrices A and B by:

$$A = \begin{bmatrix} 0.03 & 58.9 \\ 5.31 & -6.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.780 & 0.564 \\ 0.913 & 0.659 \end{bmatrix}$$

   (a) [3 points] Use MATLAB (just so that the values are as close to the true values as possible) to obtain precisely the reduced forms of A and B that result from Gaussian elimination. Store these in MATLAB as rA and rB. From the written homework, enter the approximate reduced forms that you obtained using 3 digit arithmetic. Store these in MATLAB as cA and cB. (these stand for real A (rA) and computed A (cA))

   (b) [2 points] Compute the matrices $\delta A = rA - cA$ and $\delta B = rB - cB$. Store these as dA and dB in MATLAB.

   (c) [2 points] Use the backslash operator to find the true solutions to $A\vec{x} = \vec{b}$, where $\vec{b} =< 59.2, 47.0 >$, and to $B\vec{x} = \vec{c}$ where $\vec{c} =< .218, .254 >$.

   (d) [2 points] Compute the condition number of each matrix.

   (e) [[3 points] Compute $\frac{\|\delta\vec{x}\|}{\|\vec{x}\|}$, $\frac{\|\delta\vec{b}\|}{\|\vec{b}\|}$, $\frac{\|\delta A\|}{\|A\|}$, and the corresponding values for the relative errors for $B\vec{x} = \vec{c}$ using the functions you developed in number 1. Check to see that the inequality

$$\frac{\|\delta\vec{x}\|}{\|\vec{x}\|} \leq \kappa(A)\left(\frac{\|\delta A\|}{\|A\|} + \frac{\|\delta\vec{b}\|}{\|\vec{b}\|}\right)$$

   from lecture is satisfied.

3. [5 points] Write a function that takes in an arbitrary upper triangular matrix U (so it is also square) and a vector $\vec{b}$ and returns the solution to $U\vec{x} = \vec{b}$ obtained by **back-substitution**.

(in other words, the solution is obtained in the same way we have been getting the final solution to a system in class – from the upper triangular matrix that is the end product of Gaussian elimination.. the last term x_n is easy to find, and then you use it in the second to last equation for x_{n-1}, and then use those two to find x_{n-2} etc.)

Be sure to check that it performs correctly by comparing to the solution found by MATLAB's backslash operator.