Homework Set # 8 – Math 371 – Fall 2009
Quiz Date: 11/24

1. Convert the problem
$$y'' - 0.1(1-y^2)y' + y = 0$$
with $y(0) = 1$ and $y'(0) = 1$ to a first order system.

2. Suppose that
$$\frac{dy}{dt} = f(t, y(t)) \ .$$
   (a) Use a Taylor Series expansion with a remainder term to show that
$$y(t_n + h) = y(t_n) + hf(t_n, y(t_n)) + \frac{h^2}{2}\frac{df}{dt}(t_n, y(t_n)) + \frac{h^3}{6}\frac{d^2 f}{dt^2}(\xi, y(\xi)) \ ,$$
   where $\xi \in (t_n, t_n + h)$.

   (b) Write an algorithm for generating approximates $y_0, y_1, y_2, \ldots$ using the expansion from part (a). Show that it has local truncation error of order h^3.

   (c) Apply this method to the ODE (method is referred to as Taylor's method of order 2)
$$\frac{dy}{dt} = t + y$$
   with $y(0) = 0$.

3. Find the ranges for h that yield stability for the implicit trapezoid method
$$y_{n+1} = y_n + \frac{h}{2}(f(t_n, y_n) + f(t_{n+1}, y_{n+1})) \ ,$$
applied to the problem $y' = \lambda y$, with $y(0) = y_0$.

4. Derive a three-step implicit method that is accurate on polynomials up to degree three.

5. Show that the modified Euler Method can be constructed as a predictor corrector method that uses Euler's (explict) method as the predictor, and some implicit method as the corrector (identify which one).