Homework Set # 7 – Math 371 – Fall 2009
Quiz Date: none! practice for exam 2

1. Consider the integral $\int_1^2 x\cos(\pi x)\,dx$.
   (a) Compute the integral exactly by hand.
   (b) Approximate the integral using the Midpoint rule, Trapezoid rule, Simpson's Rule, composite Simpson's rule, and the extrapolated Simposon's rule. Compute the error for each rule.
   (c) Finally, chop the interval into 2 equal pieces and do Simpson's rule on each piece. By what factor is the error improved? How does the answer compare with using composite Simpson's rule on each piece?

2. Approximate the following integral using your favorite quadrature rule: $\int_1^2 \frac{\cos(\pi x)}{\sqrt{x}}\,dx$. Notice that we cannot integrate this by hand formulaically, so there is no way to check how well we've approximated the integral. How could you ensure that the value you've obtained is fairly accurate? Can you think of a way to estimate the error?

3. Show that the integral of the Hermite interpolating polynomial

$$P_k(s) = \frac{3hs^2 - 2s^3}{h^3}y_{k+1} + \frac{h^3 - 3hs^2 + 2s^3}{h^3}y_k + \frac{s^2(s-h)}{h^2}d_{k+1} + \frac{s(s-h)^2}{h^2}d_k$$

   over one subinterval is

$$\int_0^h P_k(s)\,ds = h\frac{y_{k+1} + y_k}{2} - h^2\frac{d_{k+1} - d_k}{12}$$

4. (by request!) Solve the following matrix problem for the least-squares approximation

$$\begin{pmatrix} 1 & 6 \\ 2 & 9 \\ 2 & 18 \end{pmatrix}\vec{\beta} = \begin{pmatrix} \frac{3}{2} \\ 3 \\ 3 \end{pmatrix}$$

   by using appropriate Householder reflections to reduce the matrix to row echelon form and then using back substitution. (do by hand! this one works out nicely)