

Homework Set # 6 – Math 371 – Fall 2009

Quiz Date: 10/29/2009

1. The average scores reported by golfers of various handicaps on a difficult par-three hole are as follows:
- | Handicap | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Average | 3.8 | 3.7 | 4.0 | 3.9 | 4.3 | 4.2 | 4.2 | 4.4 | 4.5 | 4.5 |

- (a) Construct the matrix problem $X\vec{\beta} = \vec{y}$ associated to this data assuming that you are looking for the least-squares linear approximation. Does this matrix have full rank?
 - (b) Find the associated “normal equations” - i.e. find the system $N\vec{\beta} = \vec{z}$, where $N = X^T X$ and $\vec{z} = X^T \vec{y}$. Is this problem well or ill conditioned?
 - (c) Construct the matrix problem associated to this data assuming that you are looking for the least-squares quadratic approximation, and check the rank. Then find the associated “normal equations”. Again, is this well or ill conditioned?
 - (d) Use MATLAB to solve the systems for both the linear and quadratic cases. Explain what exactly you solve, how you solve it, and why you chose that solution method.
 - (e) From the results you obtain, do you think that the underlying data set has behavior closer to linear or quadratic? Why?
2. Suppose H is a Householder reflection. Show that $H^T = H$ and $H^2 = I$.
3. Again, suppose that H is a Householder reflection, so that $H = I - \rho \vec{u} \vec{u}^T$. Fix vector $\vec{x} \in \mathbb{R}^n$. Suppose further that u is obtained by

$$\vec{u} = \vec{x} + \text{sign}(x_k) \|\vec{x}\|_2 \vec{e}_k .$$

Show that $H\vec{x} = C\vec{e}_k$ for some constant C .

4. Let $\vec{x} = [9; 2; 6]$. Use problem 3 to find the Householder reflection H that transforms \vec{x} into

$$H\vec{x} = \begin{pmatrix} -11 \\ 0 \\ 0 \end{pmatrix}$$

What Householder reflection H_2 would instead transform \vec{x} into

$$H_2\vec{x} = \begin{pmatrix} 0 \\ -11 \\ 0 \end{pmatrix} ?$$