Homework Set # 5 – Math 371 – Fall 2009
Quiz Date: 10/13/2009

1. Derive a recursive algorithm using Newton's method to calculate the p^{th} root of a positive number Q (i.e. - define the appropriate $f(x)$, and sub this particular function in to newton's method - this will give you the recursive algorthim). Use your method to calculate $2^{1/3}$. How many iterations are required to obtain 16 digits of accuracy starting with $x_0 = 1$? List your successive approximation values in a table.

2. Use the Secant method to prove that the sequence below converges to $\sqrt{Q}$ where $Q > 0$, given good starting values of x_0 and x_{-1}:

$$x_{n+1} = \frac{x_n x_{n-1} + Q}{x_n + x_{n-1}}$$

   Come up with similar formulae for $Q^{1/3}$ and $Q^{1/4}$ again using the secant method. (this is similar to # 1 - choose the appropriate $f(x)$ for each case, sub in, simplify and show that it agrees with the formula above.. then repeat to come up with formulae for the other powers of Q)

3. Use Newton's method to find the root $x = 2$ for the function $f(x) = (x - 2)^3$, using the starting guess $x_0 = 3$. Calculate the error $e_n = |x_n - 2|$ and $e_{n+1} = |x_{n+1} - 2|$. Determine whether or not you obtain that $e_{n+1} = Ce_n^2$ as claimed in the book (hint: look at $\frac{e_{n+1}}{e_n^2}$ is this is relatively constant, then it is).

4. Use IQI to find a root of the function $f(x) = x^2 - 4\sin(x)$ taking $x_0 = 1$, $x_1 = 2$, $x_2 = 3$ as starting values. Give a table of the successive approximations.