Homework Set # 3 – Math 371 – Fall 2009
Quiz date: 9/17/2009

1. Let

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & 1 \end{bmatrix}$$

What is the inverse of M? (note: rather than diving into the row reduction, try thinking first in terms of elementary row operations... make a guess and check to see if you are correct). Apply M to a generic 5×5 matrix B (i.e. compute MB). Then compute $M^{-1}(MB)$.

2. Let

$$A = \begin{bmatrix} 0 & 2 & -4 & -8 \\ 2 & 1 & 2 & 0 \\ -2 & 2 & -2 & 1 \\ 4 & -2 & -4 & 1 \end{bmatrix}$$

Compute the $LU = PA$ factorization by hand using Gaussian elimination with partial pivoting. [In other words, find L, U, and P and check to make sure you've got the right factorization in the end] Use the decomposition to find the solution to $A\vec{x} = \vec{b}$ where $\vec{b} =< 1, 2, 0, -1 >$.

3. Suppose A is an $n \times n$ matrix with LU factorization $LU = PA$. Prove that $\det(A) = \pm \det(U)$. [hint: you might need to go back to your linear algebra book and read up on determinants of triangular and permutation matrices..]

4. Because any invertible matrix can be reduced to the identity matrix via Gaussian elimination, it is easy to see that we can find an LU factorization of **any** invertible matrix A of the form $LU = PA$ via the procedure discussed in class. It is tempting to think that we would also be able to find a similar factorization $A = LU$ where L is lower triangular and U is upper triangular for any invertible matrix A. This is not the case, in general! This exercise illustrates this fact.

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

   (a) Show that A is invertible.

   (b) Show that it is impossible to have a lower triangular matrix L and an upper triangular matrix U such that $A = LU$.