

Homework Set # 2 – Math 371 – Fall 2009

Quiz date: 9/8/2009

1. Consider the system $A\vec{x} = \vec{b}$, where A is an $n \times n$ invertible matrix and $\vec{x}, \vec{b} \neq \vec{0}$ are in \mathbb{R}^n . consider the perturbed system

$$A(\vec{x} + \delta\vec{x}) = \vec{b} + \delta\vec{b}$$

Then show that

$$\frac{\|\delta\vec{x}\|_*}{\|\vec{x}\|_*} \geq \frac{1}{\kappa_*} \frac{\|\delta\vec{b}\|_*}{\|\vec{b}\|_*}.$$

2. Let the matrix A be defined by

$$A = \begin{bmatrix} .03 & 58.9 \\ 5.31 & -6.1 \end{bmatrix}$$

- (a) By hand, compute the norms $\|A\|_\infty$ and $\|A^{-1}\|_\infty$. Use this to find $\kappa_\infty(A)$.
(b) Use three digit arithmetic and Gaussian elimination to approximate the solution to $A\vec{x} = \vec{b}$, where $\vec{b} = \langle 59.2, 47.0 \rangle$.
(c) Now repeat part (b) using partial pivoting.
3. Define the matrix A by:

$$A = \begin{bmatrix} 0.780 & 0.564 \\ 0.913 & 0.659 \end{bmatrix}$$

- (a) Use Gaussian elimination **without** partial-pivoting and three-digit arithmetic to obtain a solution to $A\vec{x} = \vec{b}$, where $\vec{b} = \langle 0.218, 0.254 \rangle$.
(b) Repeat using partial-pivoting.
(c) Explain the difference between the behavior you see here and that of parts (b) and (c) of problem 2.
4. Show that the matrix system $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

and $\vec{b} = \langle 1, 3, 5 \rangle$. Has infinitely many solutions. Describe the set of solutions. Now ask MATLAB to find the solution to the system using the backslash operator. What solution is obtained? In what sense is it a good solution? In what sense is it a bad solution?

Finally, let $\vec{b} = \langle 1.1, 3, 5 \rangle$. Now how many solutions does $A\vec{x} = \vec{b}$ have? Again use the backslash operator to see what MATLAB has to say. What is happening here?