Homework Set # 1 – Math 371 – Fall 2009 Quiz date: 9/1/2009

1. If a is an approximate value for a quantity whose true value is t, and a has relative error r, prove from the definitions of these terms that $a = t(1 + r)$.

2. Let $p(x) = 1.01x^4 - 4.62x^3 - 3.11x^2 + 12.2x - 1.99$.

   (a) Show that the above polynomial can also be written in nested form as was demonstrated in lecture.

   (b) Use three-digit rounding arithmetic to evaluate p(4.62), using the form of the polynomial given above.

   (c) Use three-digit rounding arithmetic to evaluate p(4.62), using the nested form of the polynomial from part (a).

   (d) Compare the approximations in parts (b) and (c) to the true three-digit result p(4.62) = -7.61 by finding the absolute and relative errors.

3. Consider $p(x) = x^2+62.10x+1 = 0$. The roots of this equation are $x_1 = -0.1610723$ and $x_2 = -62.08390$ to seven significant digits. Use four digit rounding arithmetic to approximate the roots of p using the quadratic formula. Then compute the relative error for each approximated root. One root should be much more accurate than the other - explain why.

4. For computing the midpoint m of an interval $[a, b]$, which of the following two formulas is preferable in floating-point arithmetic? When? Why? (Give examples to illustrate your reasoning)

   (a) $m = (a + b)/2.0$

   (b) $m = a + (b - a)/2.0$

5. Let $\vec{x} \in \mathbb{R}^n$. Show that

$$\|\vec{x}\|_\infty \leq \|\vec{x}\|_2 \leq \sqrt{n}\|\vec{x}\|_\infty \ .$$

6. Suppose that $\|\cdot\|$ defines a vector norm on $\mathbb{R}^n$. Let A be an $n \times n$ invertible matrix with real entries. Show that

$$\|\vec{x}\|_* = \|A\vec{x}\|$$

   defines a norm on $\mathbb{R}^n$.

7. Show that for A defined by

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

   for any real numbers a, b, $\|A\|_1 = \|A\|_2 = \|A\|_\infty$.

8. Show that if

$$A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$$

   for any real numbers a, b, then $\|A\|_2 = \sqrt{a^2 + b^2}$.