Test Review Questions

Consider the following when preparing for the tests. Always read the book carefully and be sure you know how to do the assigned problems.

1. Know all about stories 3, 5, 7, and 9 from chapter 1. Notice that the complete answers are in section 1.3 and a game version of Let's Make a Deal is on the CD-ROM from your kit.

2. The life lessons.

3. Know (and use) all four steps when solving a problem. Know (and use) the techniques of problem solving.

4. Who are G.H. Hardy? Srinivasa Ramanujan? Fibonacci? Pierre de Fermat? Andrew Wiles? The Pythagoreans? (Hint: "mathematicians" is not a good enough answer.)

5. Be able to make progress (not necessarily solve completely) working on problems that are as difficult as problem #14 in section 1.4. Use the problem solving steps and the techniques.

6. Thompson-Boling Arena holds 24,535 people. It was a sell-out crowd, and the Vols won! Everyone was so happy that they decided to have a party every day for the next year. They decided that each person would attend the party which occurred on his or her own birthday. Show that at least one of these parties will have at least 50 people.

7. Your mom's office is having one of those jelly bean jar contests for Easter. Someone got a really big jar and filled it with jelly beans, and you enter the contest by guessing how many jelly beans are in the jar. Explain how you would come up with your guess.

8. You have a deck of cards. How many cards do you have to draw in order to be sure that you have two cards of the same number? Two of the same suit? Two of the same color?

9. The pigeon-hole principle.

10. The Fibonacci numbers, where we see them in nature, and how we compute them.

11. What's so special about $1 + \cfrac{1}{1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}}$?

12. How do we write a number as the sum of Fibonacci number? Try for 200 and 232. (Hint: 144 is a Fibonacci number.)

13. How do we win at Fibonacci Nim? Suppose that it is your turn and there are 20 sticks left and that the previous player took 2 sticks. How many do you take?

14. What is the Golden Ratio? What other topics in this course is it related to? What is its exact value (not a decimal)? What is its approximate (decimal) value?

15. What is a prime number? How can we tell that a number is (or isn't) prime? Is 51 a prime? Is 71 a prime?

16. The prime factorization of the natural numbers. What is it? What does it mean? Justify that it is true. What is the prime factorization of 270?

17. How many primes are there (answer: infinitely many)? How do we *prove* that? (This means: understand the proof.) How can you convince me that there must be a prime number larger than 600? (Note: you can't write out — or multiply out — the quantity you'd need and show all its digits. It's just too big. But with judicious use of ... you can get your point across.)

18. Fermat's Last Theorem. Why is it so famous?

19. The Twin Prime Question and the Goldbach Question. Why are they so famous?

20. List some values of n for which $2^n - 1$ is a prime number. List some values of n for which it is not a prime number.

21. What is a natural number? A rational number? An irrational number? A real number? Can you give examples of each type? What about of a rational number which is not a natural number? A real number which is not a rational number?

22. Be able to explain the following: If we have a natural number n and we know that 2 divides evenly into n^2, then it *must* be the case that 2 divides evenly into n.

23. Show that $\sqrt{2}$ is not a rational number. Show that $\sqrt{3}$ is not a rational number.

24. Show that the product of two rational numbers is another rational number.

25. Explain the idea of "no holes, no neighbors" for the real numbers. There is no next real number.

26. Decimal expansions. How can you tell if a decimal expansion of a number describes a rational or an irrational number?

27. What are periodic decimals? What steps do you take to rewrite them as fractions? What is 0.22999999999999... when written as a fraction?

28. When you consider all the real numbers, are there more rationals or irrationals?

29. Know all about the dodgeball game. Explain the strategy for any size board.

30. What are the natural numbers? The rational numbers? The real numbers?

31. Who is Georg Cantor?

32. State a completely accurate definition of *one-to-one correspondence.*

33. State a completely accurate definition of what it means for *two sets to be equally numerous.*

34. What does the word *cardinality* mean?

35. For each pair of sets, determine whether they are equally numerous. If they are equally numerous, give a one-to-one correspondence. If they are not, explain why you are sure.

    (a) $\{1, 2, 3, 4, 5, \ldots\}$ and $\{2, 4, 6, 8, 10, \ldots\}$
    (b) $\{\pi, e, \sqrt{2}\}$ and $\{7, 13, 42\}$
    (c) $\{1, 2, 3, 4, 5, \ldots\}$ and $\{5, 6, 7, 8, 9, 10, \ldots\}$
    (d) $\{1, 10, 100, 1000\}$ and $\{7, 77, 777\}$
    (e) $\{1, 2, 3, 4, 5, \ldots\}$ and $\{\pi, \pi^2, \pi^3, \pi^4, \pi^5, \ldots\}$
    (f) $\{1, 2, 3, 4, 5, \ldots\}$ and $\{1, 2, 3, 5, 8, 13, 21, 34, \ldots\}$
    (g) $\{1, 2, 3, 4, 5, \ldots\}$ and $\{2, 3, 5, 7, 11, 13, 17, 23, 29, \ldots\}$
    (h) $\{1, 2, 3, 4, 5, \ldots\}$ and $\{\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots\}$
    (i) The natural numbers and the integers.
    (j) The natural numbers and the rational numbers.
    (k) The integers and the rational numbers.
    (l) The natural numbers and the real numbers.
    (m) The natural numbers and the real numbers whose decimal expansion consists only of twos and threes.
    (n) States in the United States and members of the US Senate.
    (o) States in the United States and current governors.

36. Neyland Stadium seats 104,079 people. Is there a one-to-one correspondence between the people in a sell-out crowd and their birthdays? Is there a one-to-one correspondence between the people in a sell-out crowd and the day that they die?

37. When Dr. Szczepański taught Math 110 in the Fall of 2003 there were 37 students enrolled in the class. It met in Ayres 205. Some days (especially when there was a test), there would be students sitting on the floor. Explain why this means that there were fewer than 37 desks in A205.

38. Consider the set of people living in the United States and the set of phone numbers in service. Is the pairing which matches people to their phone numbers a one-to-one correspondence?

39. Given a list of decimal numbers (real numbers), find a number which is certainly not on the list.

40. The Hotel Cardinality (sometimes called the Hilbert Hotel).

41. From the textbook: section 3.2: 16–18, 21.

42. From the textbook: section 3.3: 12.

43. Be able to state the Pythagorean Theorem in words.

44. Given the lengths of two sides of a right triangle, use the Pythagorean Theorem to find the length of the other side.

45. Explain why the "puzzle proof" shows that the Pythagorean Theorem is true.

46. State the Art Gallery Theorem completely accurately and in your own words.

47. Know how to apply the Art Gallery Theorem.

48. Know the story about Steve Fisk.

49. Be able to choose the best-suited points to put the cameras in the Art Gallery.

50. Know how many and where to place the cameras in a comb shaped gallery.

51. (a) Sketch a gallery with 18 vertices which needs 6 cameras.
    (b) Sketch a gallery with 12 vertices which needs 4 cameras.
    (c) Sketch a gallery with 12 vertices which needs only 3 cameras.

52. (a) How many cameras will you need for a gallery with 8 vertices?
    (b) Can you draw a gallery with 12 vertices which needs 5 cameras? Why or why not?
    (c) Draw a gallery with 6 vertices which needs 1 camera (or explain why it's impossible).
    (d) Draw a gallery with 6 vertices which needs 2 cameras (or explain why it's impossible).
    (e) Draw a gallery with 6 vertices which needs 3 cameras (or explain why it's impossible).
    (f) Same question as above, but for a gallery with 10 vertices for 1, 2, 3, or 4 cameras.

53. Which of our problem solving techniques did we employ in the proof of the Art Gallery Theorem?

54. Be able to divide the museum into triangles and label/color the vertices with their respective labels.

55. Sketch a Golden Rectangle.

56. Given any rectangle, determine whether or not it is a Golden Rectangle.

57. What happens if you start with a Golden Rectangle and remove the largest square from it?

58. Name a work of art or architecture which contains a Golden Rectangle.

59. Given the base (long side) of a Golden Rectangle, find its height (short side).

60. Given the height (short side) of a Golden Rectangle, find its base (long side).

61. Be able to find the area of a Golden Rectangle given either its base or its height.

62. Be able to draw the Golden Spiral and find its center.

63. Define *symmetry*.

64. What is *rigid symmetry*?

65. Know what *symmetry of scale* means. What are *supertiles*?

66. What shape is the basic building block of the Pinwheel Pattern?

67. Produce supertiles of the Pinwheel Pattern from five smaller tiles.

68. Given an illustration of the Pinwheel Pattern, outline a five-tile supertile.

69. Given an illustration of the Pinwheel Pattern, show how five five-tile supertiles can be combined to form a 25-tile supersupertile.

70. What defines 1-Dimensional Space? 2-Dimensional Space? 3-Dimensional Space?

71. Know the degrees of freedom associated with each space. How many pieces of information do you need to find a point in each space?

72. Is there a Fourth Dimension? Explain your answer.

73. Be able to show a cube in 1,2, and 3-Dimensional space. Know the number of vertices, edges, and faces for each cube.

74. Draw a cube in the Fourth Dimension.

75. The difference between a polygon and a polyhedron.

76. The definition of a regular polygon.

77. The definition of a regular polyhedron.

78. The names of the five Platonic Solids.

79. Given a drawing of a Platonic Solid, identify it by name.

80. How many vertices, faces, and edges each Platonic Solid has.

81. The concept of duality. What is the dual of each Platonic Solid?

82. Euler characteristic equation.

83. How the Euler characteristic applies to planar graphs.

84. Given two of the following: number of vertices, number of faces, and number of edges. Compute the third quantity.

85. Let s stand for the number of sides on a regular solid, and let c stand for the number of edges leaving each vertex. Given ("given" in this case means "you don't have to memorize because it would be provided.") the three inequalities: $s \geq 3$, $c \geq 3$, and $\frac{2}{c} + \frac{2}{s} > 1$, make a list of allowable values of s and c. Which Platonic Solids correspond to each set of values? Why does this list say about the number of possible Platonic Solids?

86. What does it mean for two objects to be *equivalent by distortion*?

87. How do you take off a rubber vest without taking off your jacket?

88. What happens if you turn an innertube with a hole inside-out?

89. A ring passes through two holes in a piece of silly putty. Is this equivalent by distortion to the ring passing through only one hole in the silly putty?

90. Explain how a crawling bug con tell whether two line drawings in the plane are equivalent by distortion.

91. Take the letters of the alphabet and divide them into categories so that you group together letters which are equivalent by distortion.

92. Can the crawling bug tell the difference between a sphere and a torus?

93. How can you show that a sphere and a torus are not equivalent by distortion?

94. How many sides does a cylinder have? How many edges? What happens if you cut it down the middle?

95. What is a Möbius strip? How do you make one? How many sides does it have? How many edges? What happens if you cut it down the middle? What if you cut it a third of the way over from the edge?

96. Given a square or rectangle together with instructions on how to glue the edges together, decide whether the shape is a cylinder, a Möbius strip, a torus, or a Klein bottle.

97. Given an unfolded, unglued Möbius strip and the initial path of a crawling bug, describe where the bug will end up if it keeps crawling along the same path.

98. What is a Klein bottle? List one of more interesting facts about this shape.

99. You have the map of Tennessee which shows all the counties in the state. What is the most colors that you will need to color in this map so that no counties which share a border are colored the same color?

100. Draw a map on a Möbius strip (pretend that it is made of clear plastic or that your ink is really wet so the colors bleed through the paper) which needs five colors. What about one which needs six colors?

101. Place in order from least likely to most likely: chance of getting *heads* when flipping a coin, spinning a coin, or balancing a coin on edge then letting it fall.

102. Know the rules of the *Let's Make a Deal* game and what your best strategy is (and why). Review the full explanation from section 1.3.

103. Be able to estimate (and place in order) the probabilities of several everyday events. Place in order: the chance of being involved in a motor vehicle accident on Cumberland Avenue, the chance that a Lady Vol athlete who has exhausted her eligibility will graduate from UT (hint: *very* high), the chance that UT will change the Arts and Sciences graduation requirements from two math classes to one math class.

104. Be able to give a clear, complete, and accurate definition of the *probability of an event occurring* and of *relative frequency*.

105. Be able to give a clear, complete, and accurate statement of the *Law of Large Numbers*, to restate it in your own words, and to give an example which shows that you understand what it means.

106. If you roll one die, what is the probability of getting a two? Of not getting a two? Of getting a number higher than two?

107. If you roll one die twice, what is the probability of getting no twos? If you roll one die three times, what is the probability of getting no twos? Three times? Four? Etc.

108. You have rolled one die twice, and you have gotten no twos. If you roll it again, what is the chance of not getting a two? (Careful.)

109. If you roll two dice, how many possible equally likely outcomes are there? if you roll two dice, what is the probability of getting a total of four? What is the probability of getting a total less than or equal to four? What is the probability of getting a total greater than four?

110. In Pick 3 Lotto, you choose three numbers between zero and nine. How many possible Pick Three numbers are there? If you buy one ticket, what is your chance of winning?

111. What are the equally likely outcomes if you flip a penny, a nickel, and a dime? What is the probability of getting all heads? Of getting exactly two heads? Of getting at least two heads? If you know that the dime is heads, what is the probability that you got at least two heads?

112. Explain the *It either happens or it doesn't* property. Give an example to illustrate what it means in terms of computing probability.

113. State the *Infinite Monkey Theorem* and explain what it means.

114. Suppose a freak event has a 1 in 1000 chance of happening. What is the chance that you avoid this event for a full year? For ten years? What if the event had only a 1 in 10,000 chance of happening?

115. There are approximately 27,000 students at UT. Suppose that 1 out of 100 UT students are impolite, and Emily Post (the etiquette expert) is coming to campus to test the manners of the entire student body. Unfortunately, she is not perfect. When talking to an impolite person, she correctly identifies their rudeness 80% of the time and misses it 20% of the time. When talking to a polite person, she recognizes their good manners 90% of the time and incorrectly accuses them of rudeness 10% of the time. How many UT students are impolite? How many are polite? How many students will Emily Post think are impolite? If Emily Post thinks that a UT student is rude, what is the chance that they really are?