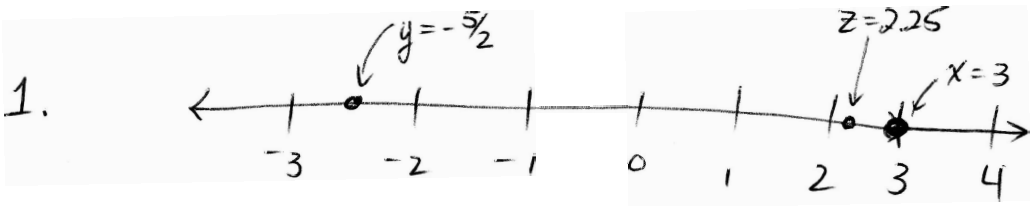
4.7:

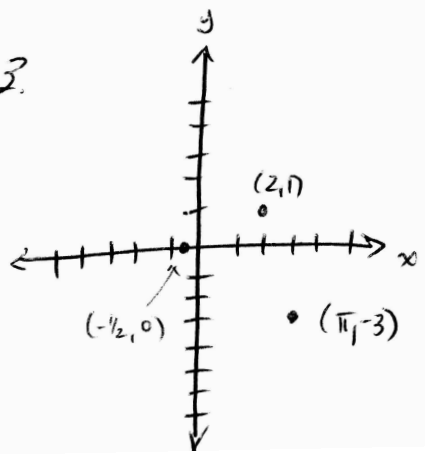
1.



need one number to identify a point in the space, so the space is one dimensional.

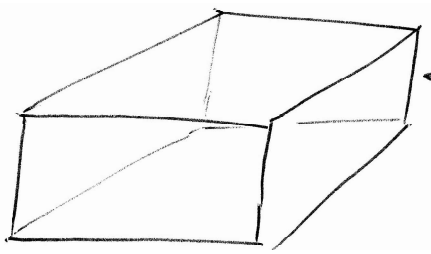
2. In a plane you can move left and right and up and down. Saying that "$x = 4$" conventionally means that "you go 4 units to the right of the origin", but there is nothing to specify how far up or down you need to go. There are infinitely many points in the plane that you can reach by first moving 4 units right and then moving various distances up or down. We need one more piece of info to get a unique point in the plane, so the dimension is 2.

3.



no! every point in the plane can be specified exactly with 2 numbers because the plane is 2-dimensional.

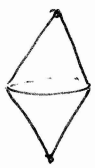
4. Get a "cube" of paper → will be 3 dimensional

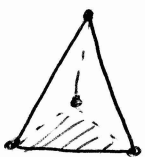


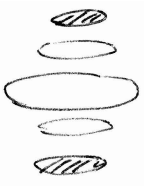
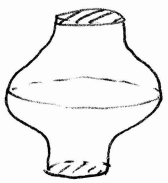
← stack of sheets of paper.

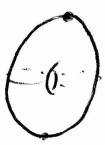
5. You would get an 18-dimensional space because to find a particular point in the stack you need 18 pieces of info: 1 to tell you which 17-dim "slice" you are in, and 17 more to find the exact point within that 17-dim slice.

7. slices → Object

(1)  →  sphere or  double cone or.

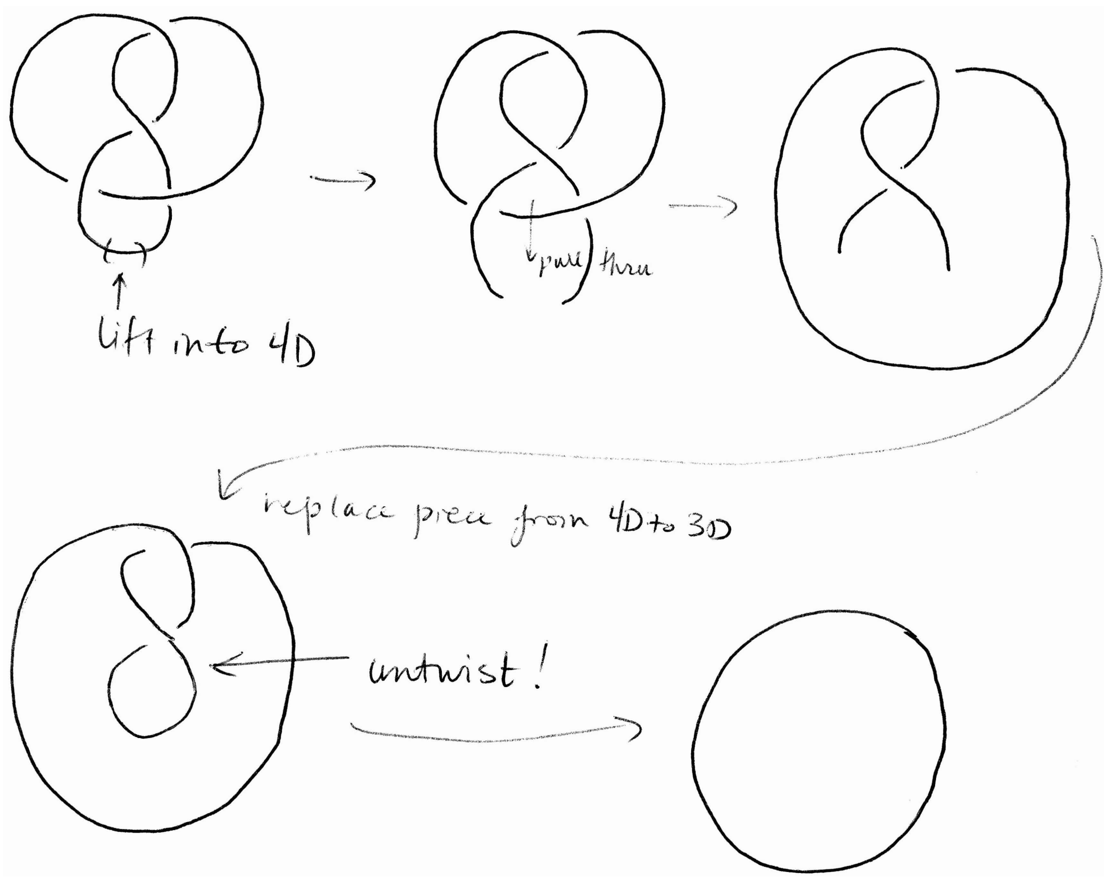
(2)  →  tetrahedron.

(3)  →  a vase-like object with closed ends

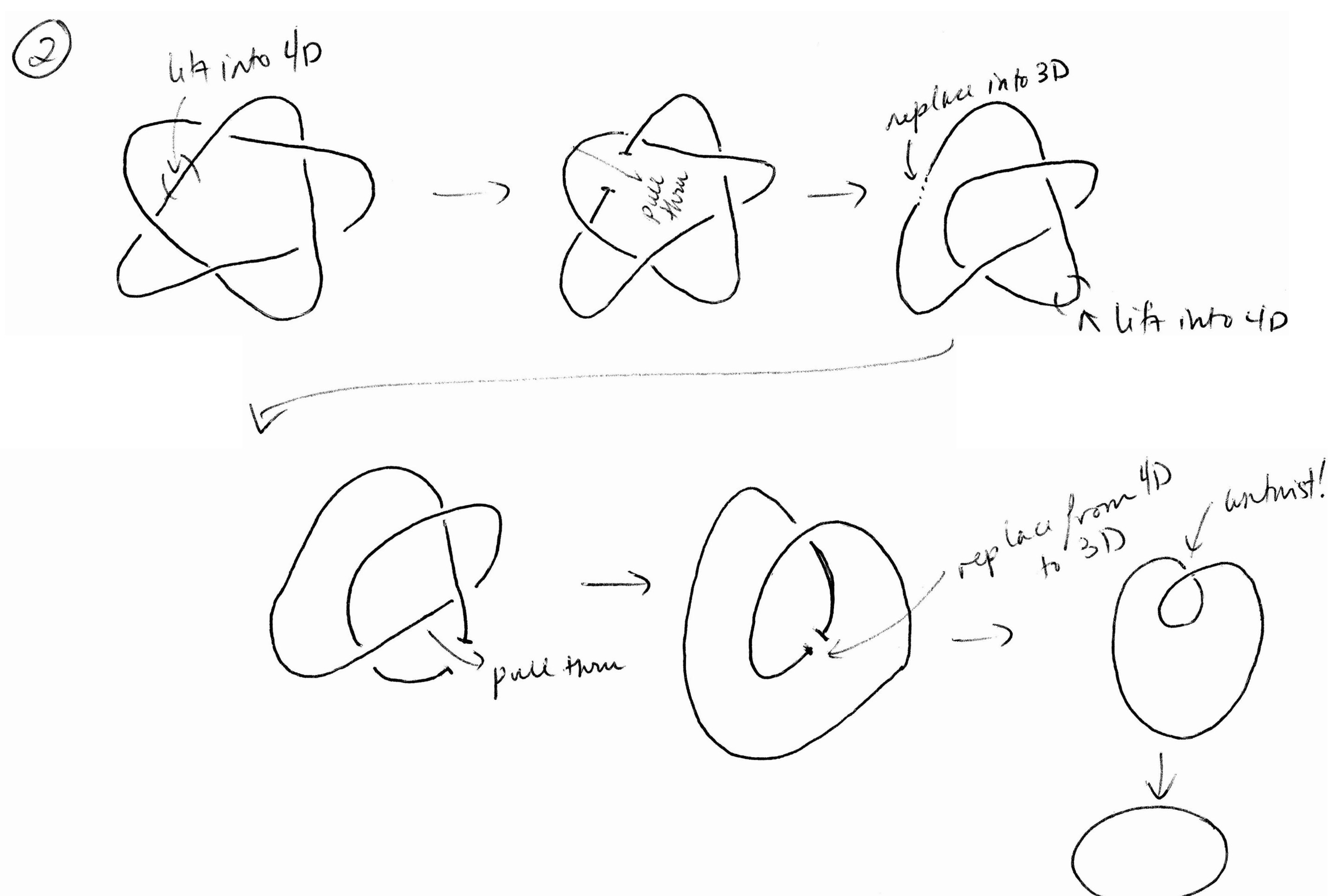
(4)  →  torus! (hollow doughnut)

12.

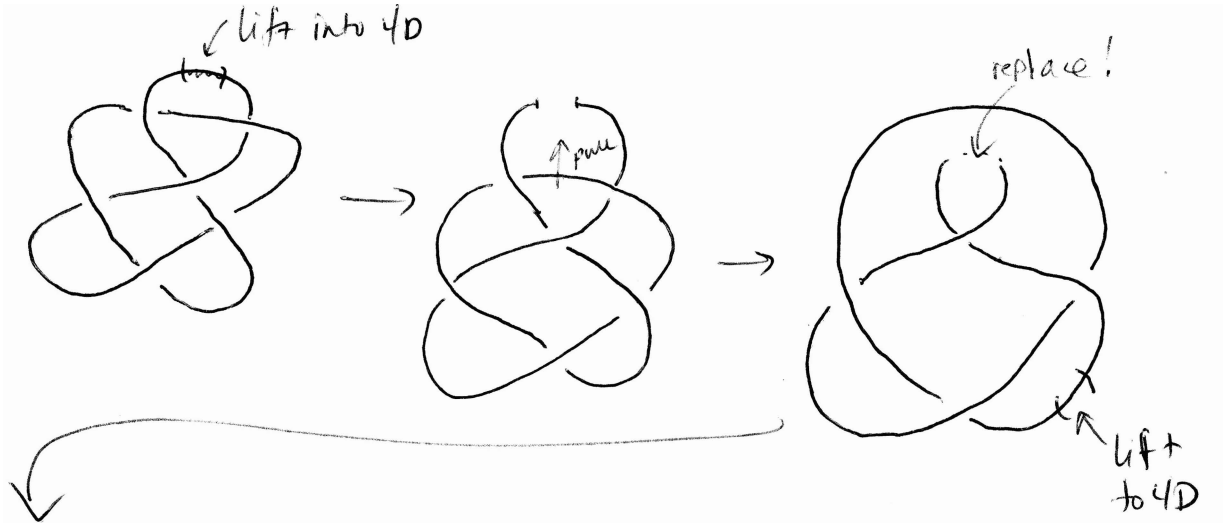
①

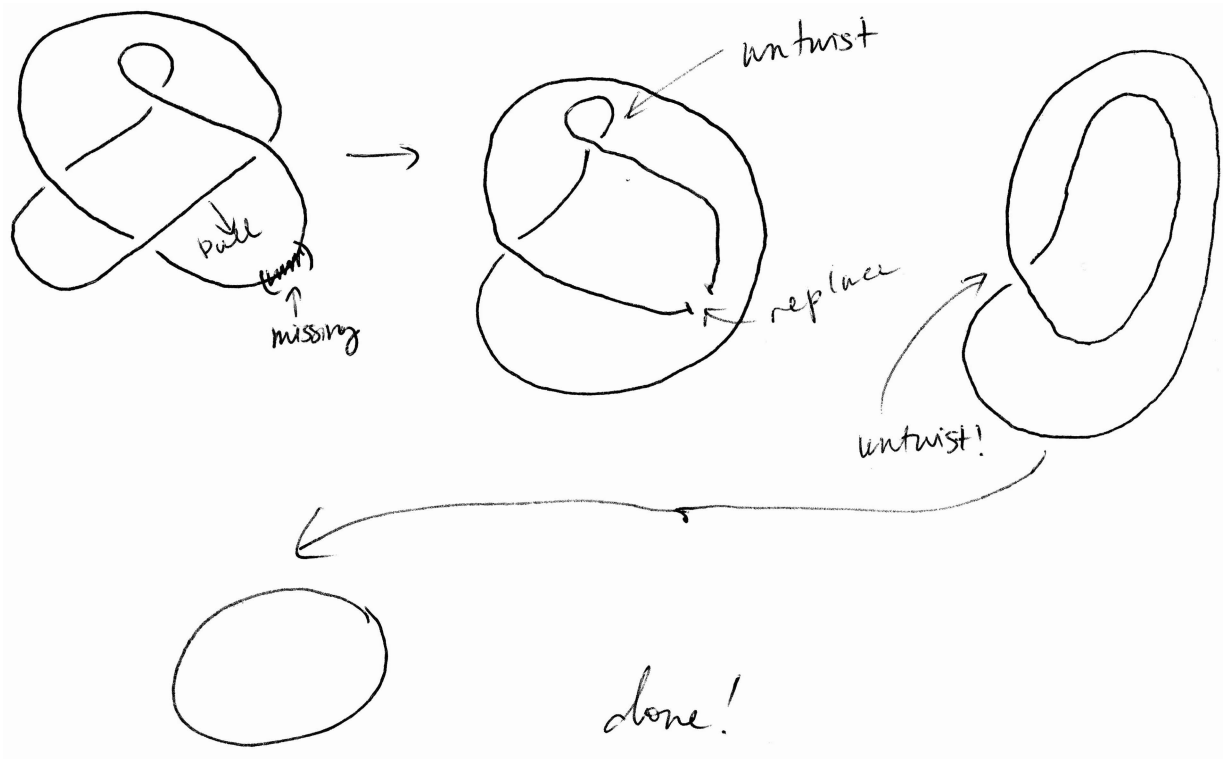

lift into 4D
pull thru
replace piece from 4D to 3D
untwist!

②


lift into 4D
pull thru
replace into 3D
lift into 4D
pull thru
replace from 4D to 3D
untwist!

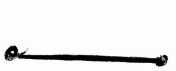
(3)

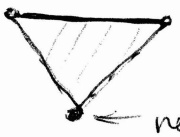

lift into 4D
pull
replace!
lift to 4D

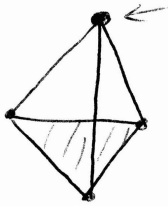

pull
missing
untwist
replace
untwist!
done!

16.

0D triangle • (1 vertex, no edges)

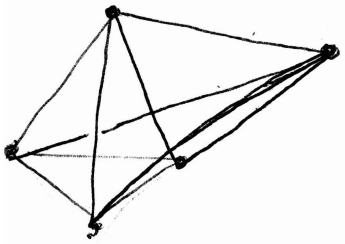
1D triangle  (2 vertices, 1 edge)

2D triangle  (3 vertices, 3 edges, 1 2D face)

3D triangle  (4 vertices, 6 edges, 4 2D faces, 1 3D face)

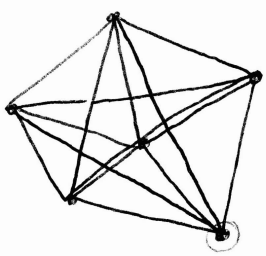
4D triangle

- get 1 new vertex,
- a new edge for every old vertex, plus old edges
- a new face for every old edge, plus old faces,
etc...



(5 vertices,
10 edges, (old edges + old verts)
10 2D faces (old edges + old faces)
5 3D faces (old 2D faces + old 3D)
1 4D face.)

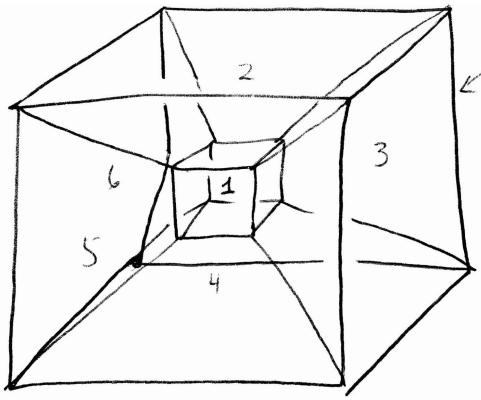
5D triangle



(6 vertices
15 edges
20 2D faces
15 3D faces
6 4D faces
1 5D faces.)

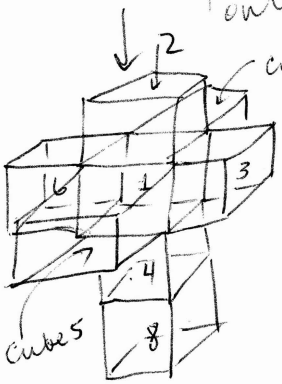
dim	verts	edges	2D faces	3D faces
1	2	1	0	0
2	3	3	1	0
3	4	6	4	1
4	5	10	10	5
5	6	15	20	15
n	$n+1$	$\frac{\text{dim} \times \text{verts}}{2} = \frac{n(n+1)}{2}$	$\frac{(n+1)(n)(n-1)}{3}$	$\frac{(n+1)(n)(n-1)(n-2)}{24}$

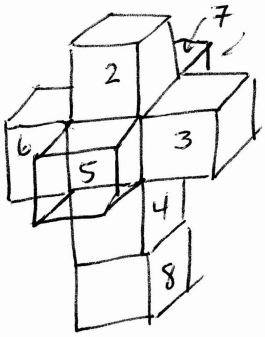
18.



4D cube (assembled)

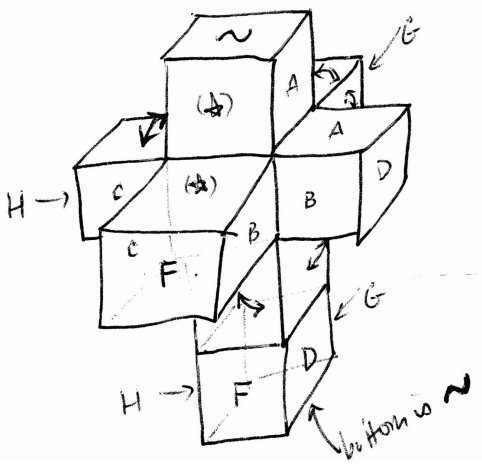
↑ outer cube = cube 8.





↓

How do I identify the faces to get the 4D cube back??



← look at picture of 4D cube above and see which faces of each 3D cube are glued together!