$$= f\,\text{curl}(\vec{F}) + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ f_x & f_y & f_z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= f\,\text{curl}(\vec{F}) + (\vec{\nabla} f \times \vec{F}) \checkmark$$

28 $\vec{r} = \langle x, y, z \rangle$ and $r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$

(a) $\vec{\nabla} \cdot \vec{r} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle x, y, z \rangle = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$

$= 1 + 1 + 1 = 3$

(b) $\vec{\nabla} \cdot (r\vec{r}) = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \left\langle x\sqrt{x^2+y^2+z^2}, y\sqrt{x^2+y^2+z^2}, z\sqrt{x^2+y^2+z^2} \right\rangle$

$= \frac{\partial}{\partial x}\left(x\sqrt{x^2+y^2+z^2}\right) + \frac{\partial}{\partial y}\left(y\sqrt{x^2+y^2+z^2}\right) + \frac{\partial}{\partial z}\left(z\sqrt{x^2+y^2+z^2}\right)$

$= \sqrt{x^2+y^2+z^2} + \frac{1}{2}x\left(x^2+y^2+z^2\right)^{-1/2}(2x) + \sqrt{x^2+y^2+z^2} + y^2\left(x^2+y^2+z^2\right)^{-1/2}$

$+ \sqrt{x^2+y^2+z^2} + z^2\left(\sqrt{x^2+y^2+z^2}\right)^{-1}$

$= \sqrt{x^2+y^2+z^2}\left(3 + (x^2+y^2+z^2)(x^2+y^2+z^2)^{-1/2-1/2}\right)$

$= \sqrt{x^2+y^2+z^2}\,(4) = 4r \checkmark$

(c) $\vec{\nabla}^2 r^3 = \vec{\nabla} \cdot \vec{\nabla}(r^3) = \vec{\nabla} \cdot \left\langle \frac{\partial}{\partial x}\left[(x^2+y^2+z^2)^{3/2}\right], \frac{\partial}{\partial y}\left[(x^2+y^2+z^2)^{3/2}\right], \frac{\partial}{\partial z}\left[(x^2+y^2+z^2)^{3/2}\right] \right\rangle$

$= \vec{\nabla} \cdot \left\langle \frac{3}{2}(x^2+y^2+z^2)^{1/2}(2x), \frac{3}{2}(x^2+y^2+z^2)^{1/2}(2y), 3z(x^2+y^2+z^2)^{1/2} \right\rangle$

$= \frac{\partial}{\partial x}\left[3x(x^2+y^2+z^2)^{1/2}\right] + \frac{\partial}{\partial y}\left[3y(x^2+y^2+z^2)^{1/2}\right] + \frac{\partial}{\partial z}\left[3z(x^2+y^2+z^2)^{1/2}\right]$