22.

$$\text{curl}(\vec{F}+\vec{G}) = \text{curl}\left(\langle F_1+G_1, F_2+G_2, F_3+G_3\rangle\right)$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1+G_1 & F_2+G_2 & F_3+G_3 \end{vmatrix} = \left[\left(\frac{\partial F_3}{\partial y}+\frac{\partial G_3}{\partial y}\right)-\left(\frac{\partial F_2}{\partial z}+\frac{\partial G_2}{\partial z}\right)\right]\vec{i}$$

$$+\left[\left(\frac{\partial F_1}{\partial z}+\frac{\partial G_1}{\partial z}\right)-\left(\frac{\partial F_3}{\partial x}+\frac{\partial G_3}{\partial x}\right)\right)\vec{j}$$

$$+\left[\left(\frac{\partial F_2}{\partial x}+\frac{\partial G_2}{\partial x}\right)-\left(\frac{\partial F_1}{\partial y}+\frac{\partial G_1}{\partial y}\right)\right]\vec{k}$$

$$= \left[\left(\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z}\right)\vec{i}+\left(\frac{\partial G_3}{\partial y}-\frac{\partial G_2}{\partial z}\right)\vec{i}\right]$$

$$+\left[\left(\frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x}\right)\vec{j}+\left(\frac{\partial G_1}{\partial z}-\frac{\partial G_3}{\partial x}\right)\vec{j}\right]$$

$$+\left[\left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right)\vec{k}+\left(\frac{\partial G_2}{\partial x}-\frac{\partial G_1}{\partial y}\right)\vec{k}\right]$$

$$= \text{curl}(\vec{F}) + \text{curl}(\vec{G})$$

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$$\text{curl}(f\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ fF_1 & fF_2 & fF_3 \end{vmatrix} = \left(\frac{\partial}{\partial y}(fF_3)-\frac{\partial}{\partial z}(fF_2)\right)\vec{i}$$

$$+\left(\frac{\partial}{\partial z}(fF_1)-\frac{\partial}{\partial x}(fF_3)\right)\vec{j}$$

$$+\left(\frac{\partial}{\partial x}(fF_2)-\frac{\partial}{\partial y}(fF_1)\right)\vec{k}$$

$$= \left[f_y F_3 + f(F_3)_y - f_z F_2 - f(F_2)_z\right]\vec{i} + \left[f_z F_1 + f(F_1)_z - f_x F_3 - f(F_3)_x\right]\vec{j}$$

$$+\left[f_x F_2 + f(F_2)_x - f_y F_1 - f(F_1)_y\right]\vec{k}$$

$$= f\left[(F_3)_y-(F_2)_z\right]\vec{i} + f\left[(F_1)_z-(F_3)_x\right]\vec{j} + f\left[(F_2)_x-(F_1)_y\right]\vec{k}$$

$$+\left[f_y F_3 - f_z F_2\right]\vec{i} + \left[f_z F_1 - f_x F_3\right]\vec{j} + \left[f_x F_2 - f_y F_1\right]\vec{k}$$