Answers to Even Exercises, Homework Set 6

Section 11.7 # 4 By level curves, expect local minima near $(-1, 1)$ and $(-1, -1)$. Also, expect local max near $(1, 0)$. Expect saddle points near $(-1, 0)$, $(1, 1)$, and $(1, -1)$. Finding critical points, and using the second deriv test, we see all predictions are true.

44 Cost function is $C(x, y, z) = 5xy + 2(xz + yz)$, and constraint is that we have a fixed volumed V (here V represents a given constant value), so $xyz = V$. The dimensions of the aquarium which minimize the cost are $x = y = (\frac{2}{5}V)^{1/3}$, and $z = V^{1/3}(5/2)^{2/3}$.

46 Let x = length of north and south walls, y = length of the east and west walls, and z = height of the building. Then the heat loss is:

$$h(x, y, z) = 6xy + 16xz + 20yz$$

subject to the constraint

$$xyz = 4000$$

(a) solving the constraint for z and subbing into h, we get $h(x, y) = 6xy + 80000/x + 64000/y$. We also have $z \geq 4$ implies $xy \leq 1000$, or in other words, $y \leq 1000/x$. Since $x \geq 30$ and $y \geq 30$, the domain of h is: $\{(x, y)|x \geq 30, 30 \leq y \leq 1000/x\}$.

(b) The absolute minimum of h is $h(30, 30) = 10200$ and so the dimensions of the building that minimize heat loss are walls 30 m in length and 40/9 m tall.

(c) From (b) the only critical point of h, which gives a local minimum, is approximately $h(25.54, 20.43) \approx 9396$. So a building of volume 4000 m^3 with dimensions $x \approx 25.54$ m, $y \approx 20.43$ m, and $z \approx 7.67$m has the least amount of heat loss possible.

48 First write P as a function of q and r only, using the constraint. Now the key here is to realize that you are on a bounded domain: you can only have $q \geq 0$, $r \geq 0$ and $q + r \leq 1$, since q and r represent fractions of the population. So, you need to check for critical points that lie in the domain AND on the boundary of the domain.

Section 11.8# 18 Max value of f is $f(-2, \pm 2\sqrt{3} = 47$, and the minimum value is $f(1, 0) = -7$.

36 Same answer as # 44 above.

38 V(x,y,z) = xyz, subject to constraints $g(x, y, z) = xy + yz + xz = 750$ and $h(x, y, z) = x + y + z = 50$. The maximum of f is $f(\frac{1}{3}(50 + 10\sqrt{10}), \frac{1}{3}(50 - 5\sqrt{10}), \frac{1}{3}(50 - 5\sqrt{10})) = \frac{1}{27}(87,500 + 2500\sqrt{10})$, and the minimum of f is $f(\frac{1}{3}(50 - 10\sqrt{10}), \frac{1}{3}(50 + 5\sqrt{10}), \frac{1}{3}(50 + 5\sqrt{10})) = \frac{1}{27}(87,500 - 2500\sqrt{10})$. (notice here we cannot have $x = y = z$, I have chosen here x to be the distinct side.. in other words, $x \neq y$ and $x \neq z$.)