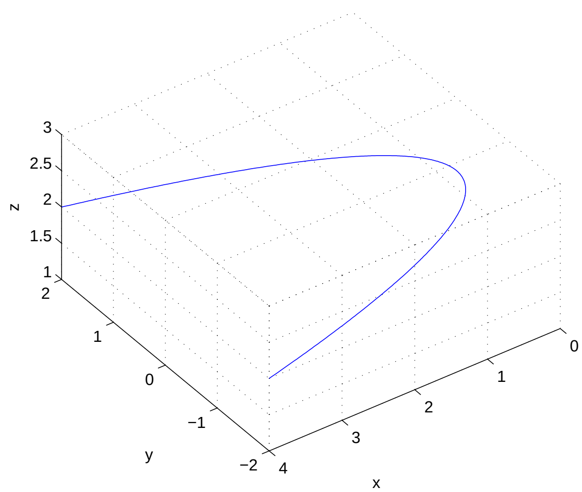
Answers to Even Exercises, Homework Set 10

Section 10.1 #2 domain $= (-3, -2) \cup (-2, 3)$

#10 This is the same as $x = y^2, z = 2$, so the graph is



#18 II

#20 I

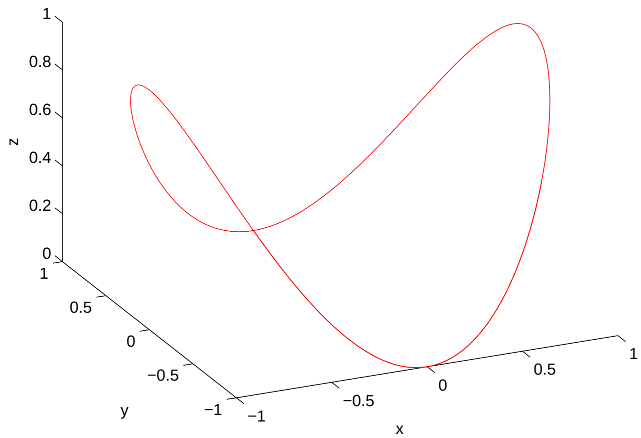
#22 III

#24 $\vec{r}(t) =< \sin(t), \cos(t), \sin^2(t) >$ satisfies:

(1) $z = x^2$ because subbing in for x(t) and z(t) given above we get $\sin^2(t) = (\sin(t))^2$

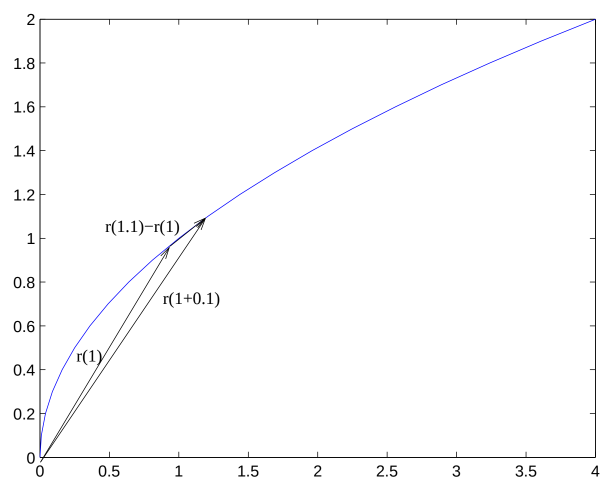
(2) $x^2 + y^2 = 1$ because again subbing in we get $(\sin(t))^2 + (\cos(t))^2 = 1$

The curve is shown as:

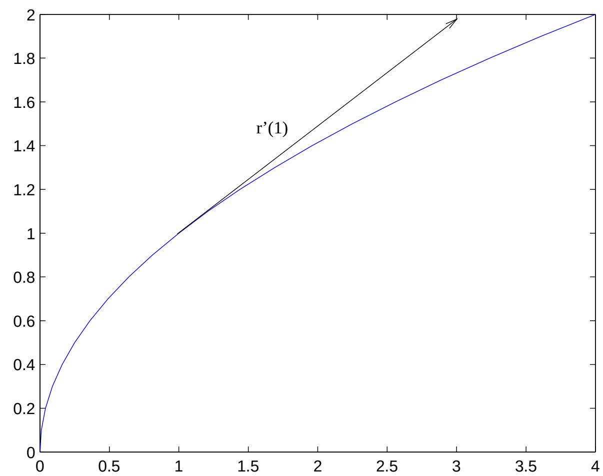


#38 Do not collide, but do intersect at $(2, 4, 8)$ and $(1, 1, 1)$.

Section 10.2 # 2 (a)



(b) $\frac{d\vec{r}}{dt}(1) =< 2, 1 >$, and $\frac{\vec{r}(1.1)-\vec{r}(1)}{.1} =< 2.1, 1 >$. These vectors are very close to one another because the second one is the difference quotient with $h = .1$, and the first is the limit of the difference quotient as $h \to 0$. Since $h = .1$ is near zero, the resulting vectors are very close.



\# 32 $\frac{7}{3}\vec{i} + \frac{16}{15}\vec{j} - \frac{3}{\pi}\vec{k}$

\# 46 If $\vec{r}(t)$ and $\vec{r}'(t)$ are always orthogonal, then their dot product is zero for all t: $\vec{r}(t) \cdot \vec{r}'(t) = 0$. If we werite this out, we see:

$$x(t)x'(t) + y(t)y'(t) + z(t)z'(t) = 0$$

this is the same as:

$$\frac{d}{dt}\left[\frac{1}{2}(x(t))^2 + \frac{1}{2}(y(t))^2 + \frac{1}{2}(z(t))^2\right] = 0$$

If we integrate the last equation, we get:

$$x^2 + y^2 + z^2 = C$$

for some constant C, which is the equation of a sphere at the origin. This shows that every point on our curve $< x(t), y(t), z(t) >$ lies on a sphere centered at the origin.