Homework Set # 4 – Math 435 – Summer 2013

1. Solve the heat equation (i.e. - the diffusion equation) $4u_{xx} = u_t$ on a rod of length 2 if $u(x,0) = sin(\frac{\pi x}{2})$ and $u(0,t) = 0 = u(2,t)$. (no need to derive the solution here, just give it)

2. Solve the wave equation $3u_{xx} = u_{tt}$ for a clamped string of length $l = 1$ (so $u(0,t) = 0 = u(1,t)$) such that $u(x,0) = 2\sin(\pi x)\cos(\pi x)$ and $u_t(x,0) = 0$. [hint: use a double angle identity from trig - and again, no need to derive it, just give it!]

3. Strauss Exercise 4, pg 89 (solve by seperation of variables, in the same way that we did in class)

4. Strauss Exercise 6, pg 89 (again, solve by sep of variables)

5. Strauss Exercise 1, pg 92.

6. Show that IF $U(x)$ is a (steady-state) solution to $U_{xx} = 0$ on $(0,l)$ with

$$U(0) = g$$
$$U(l) = h$$

for some fixed constants g, h,and IF $\tilde{u}$ is a solution to $\tilde{u}_{xx} = \tilde{u}_t$ on $(0,l)$ with

$$\tilde{u}(0,t) = 0$$
$$\tilde{u}(l,t) = 0$$

where $\tilde{u}(x,0) = f(x) - U(x)$, THEN $u(x,t) = \tilde{u}(x,t) + U(x)$ solves $u_{xx} = u_t$ where

$$u(0,t) = g$$
$$u(l,t) = h$$

and $u(x,0) = f(x)$.

[**NOTE: The point of this problem is that it allows us to solve BVP's with nonhomogeneous boundary conditions by building a solution from the homogeneous b.c. problem and the corresponding steady-state problem... Notice that the seperation of variables technique breaks down if we have inhomogeneous b.c.'s]

7. Solve problem 8 from section 5.1 of Strauss using exercise 6 above.

8. A string (with density $\rho = 1$ and tension $T = 4$) with fixed ends at $x = 0$ and $x = 10$ is hit by a hammer so that $u(x,0) = 0$ and

$$\frac{\partial u}{\partial t}(x,0) = \begin{cases} V & \text{if } x \in [-\delta + 5, \delta + 5] \\ 0 & \text{otherwise .} \end{cases}$$

Find the height of the string $u(x,t)$ for all $x \in (0,10)$ and all $t > 0$. (Your answer WILL be a bit messy...)

9. Do problems 5a and 6a from section 5.1 of Strauss. Recall that we found the Fourier sine series for $f(x) = x$ on $(0,l)$ in class and it's in the book. You do not need to rederive it.

10. Problem 15 section 5.2 of Strauss.

11. (a) Find the full Fourier series expansion of e^x on $(-2,2)$.

(b) Use MATLAB to plot the approximation

$$f(x) \approx \frac{1}{2}A_0 + \sum_{n=1}^{N}(A_n \cos(n\pi x/l) + B_n \sin(n\pi x/l))$$

for $N = 3, 5, 10, 100$, each one plotted along with a plot of the actual function $f(x) = e^x$. (all of these plots should only be over the interval [-2,2] - and make sure you label each curve)

(c) Now plot (just) the approximate Fourier series for $x \in [-10, 10]$ with $N = 10$. What do you notice? Explain what you see.