Department of Mathematics
The University of Tennessee

Fall Semester 2006
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Math 241 : Calculus and Analytic Geometry III
Quiz #6 (October 5, 2006)

Name: (please print neatly) ________________________________

Class Time: ____________

1. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1+t}$, $y = 2 + \frac{1}{3}t$. The temperature function satisfies $T_x(2, 3) = 4$, and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?

   We can use the chain rule to solve this problem, since:

   $$T_t = T_x x_t + T_y y_t$$

   The problem says that $x_t = .5(1+t)^{-.5}$ and $y_t = \frac{1}{3}$, so when $t = 3$, $x_t = .25$ and $y_t = \frac{1}{3}$. Also since $t = 3$ says that $x(3) = 2$ and $y(3) = 3$, We know from above that when $t = 3$, $T_x = 4$ and $T_y = 3$. Plugging in:

   $$T_t = 4 * (.25) + 3 * (\frac{1}{3}) = 2 \text{ degrees Celsius per unit distance}$$

2. Find the directional derivative of the function at the point $(3, 4)$ in the direction of the vector $\langle 4, -3 \rangle$ if

   $$f(x, y) = 1 + 2x\sqrt{y}$$

   The directional derivative is given by:

   $$D_{\vec{u}} f(3, 4) = \nabla f(3, 4) \cdot \vec{u}$$

   and $\vec{u}$ is a unit vector.

   First find the unit vector $\vec{u}$ in the direction of our direction vector.

   $$|\langle 4, -3 \rangle| = 5 \qquad \vec{u} = \langle 4/5, -3/5 \rangle$$

   Also the gradient of f is $\langle 2\sqrt{y}, \frac{x}{\sqrt{y}} \rangle$, so the directional derivative at $(3, 4)$ is

   $$\langle 4, 3/2 \rangle \cdot \langle 4/5, -3/5 \rangle = 16/5 - 9/10 = 23/10$$