3. Let $f(x,y) = -2x^2 - 7y^2$.

(a) [12 points] Find all local maxima, minima and saddle points for f. Does f have an absolute maximum and/or minimum? If so, where do they occur?

crit pts: $\left.\begin{array}{l} f_x(x,y) = -4x = 0 \\ f_y(x,y) = -14y = 0 \end{array}\right\} \Rightarrow (0,0)$ only crit. pt.

$f_{xx}(x,y) = -4$, $f_{yy}(x,y) = -14$ $f_{xy}(x,y) = 0$

$\Rightarrow D(0,0) = (-4)(-14) - 0^2 > 0 \Rightarrow$ max or min @ $(0,0)$

$f_{xx}(0,0) = -4 < 0 \Rightarrow$ conc. down $\Rightarrow$ local max @ $(0,0)$.

Since $f(0,0) = 0$ + $f(x,y) < 0$ if $(x,y) \neq (0,0)$ have an absolute max at $(0,0)$ + no absolute min.

(b) [10 points] Use Lagrange multipliers to find any maxima and minima of f where now we add the constraint that (x,y) satisfies $x^2 + y^2 = 1$.

$g(x,y) = x^2 + y^2$

Solve: $\vec{\nabla} f = \lambda \vec{\nabla} g$

$\langle -4x, -14y \rangle = \lambda \langle 2x, 2y \rangle$

$\Rightarrow \begin{cases} -4x = \lambda 2x \\ -14y = \lambda 2y \\ x^2 + y^2 = 1 \end{cases}$ $\Rightarrow$ either $\lambda = -2$:

$\Rightarrow -14y = -4y \Rightarrow y = 0$

and so $0^2 + x^2 = 1 \Rightarrow x = \pm 1$

OR

$x = 0$:

$\Rightarrow 0^2 + y^2 = 1 \Rightarrow y = \pm 1$ and $\lambda = -7$.

extrema are at $(0, \pm 1)$ and $(\pm 1, 0)$

$f(0, \pm 1) = -7 \leftarrow$ min value of f on $x^2+y^2=1$

$f(\pm 1, 0) = -2 \leftarrow$ max value of f on $x^2+y^2=1$

(c) [8 points] What are the absolute maximum and minimum values of f on the entire closed bounded domain $x^2 + y^2 \leq 1$? (If they exist)

putting (a) + (b) together, we see

crit pt. on interior of $x^2+y^2 \leq 1$ $\longrightarrow$ $f(0,0) = 0 \leftarrow$ abs. max value of f on $x^2+y^2 \leq 1$

$f(0, \pm 1) = -7 \leftarrow$ abs. min value of f on $x^2+y^2 \leq 1$

min value on boundary of $x^2+y^2 \leq 1$.