1. Suppose you are hiking on a hill whose surface is described by $f(x, y) = 9 - x^2 - y^2$, and you are at the point on the hill corresponding to $(x, y) = (0, 3)$.

(a) [8 points] What slope would you encounter if you walk directly in the x-direction from $(0, 3, 0)$?

in the x-direction, the slope of f is given by $f_x(x,y)$

so at $(0,3)$ slope $= f_x(0,3)$

$f_x(x,y) = -2x$

$f_x(0,3) = \underline{0}$

(b) [8 points] What slope would you encouter if you walk in the direction of $\langle 1, 2\rangle$ from $(0, 3, 0)$?

$\vec{u} = \frac{\langle 1,2\rangle}{|\langle 1,2\rangle|} = \frac{\langle 1,2\rangle}{\sqrt{1^2+2^2}} = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

$D_{\vec{u}} f(x,y) = \vec{\nabla} f(x,y) \cdot \vec{u} = \langle -2x, -2y \rangle \cdot \langle 1/\sqrt{5}, 2/\sqrt{5} \rangle$

at $(0,3,0) \rightarrow D_{\vec{u}} f(0,3) = \langle 0, -6 \rangle \cdot \langle 1/\sqrt{5}, 2/\sqrt{5} \rangle = -12/\sqrt{5}$

(c) [10 points] In what direction should you go from $(0, 3, 0)$ if you want to hike up the hill the fastest? (i.e. - in what direction do you go to encouter the greatest incline in the hill?) What is the slope you would encounter moving in this direction?

the gradient is a vector which points in the direction of greatest increase:

$\vec{\nabla} f(x,y) = \langle -2x, -2y \rangle$

so at $(0,3,0) \rightarrow \vec{\nabla} f(0,3) = \boxed{\langle 0, -6 \rangle}$

↑ direction walk in to encounter greatest slope.

the greatest slope at $(0,3,0)$ is:

$|\vec{\nabla} f(0,3)| = |\langle 0, -6 \rangle| = \sqrt{0^2 + (-6)^2} = \boxed{6}$