4. (a) [10 points] Does the following limit exist? Explain.

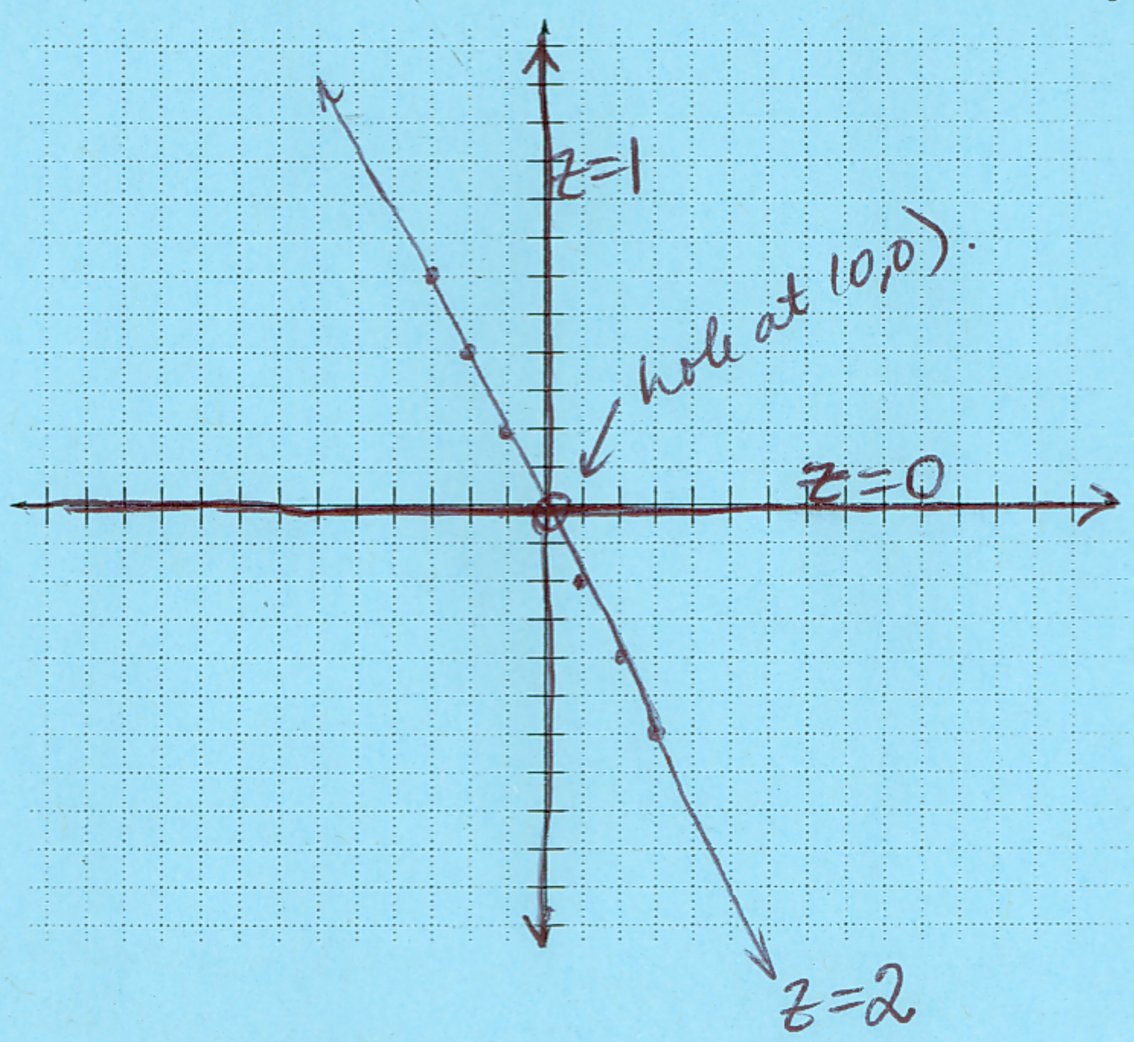
$$\lim_{(x,y)\to(0,0)} \frac{y}{x+y}$$

approaching (0,0) along the line $x=0$: $\lim_{(0,y)\to(0,0)} \frac{y}{y} = \lim_{y\to 0} 1 = 1$

approaching (0,0) along the line $y=0$: $\lim_{(x,0)\to(0,0)} \frac{0}{x} = \lim_{x\to 0} 0 = 0$

Since we get two different values for the limit by approaching (0,0) along 2 different paths, the limit does not exist.

(b) [6 points] Sketch at least 3 level curves of $f(x,y) = \frac{y}{x+y}$ ← undefined at $(x,y) = (0,0)$.



$z=0$: $0 = \frac{y}{x+y} \Rightarrow y=0$

$z=1$: $1 = \frac{y}{x+y} \Rightarrow x+y=y \Rightarrow x=0$

$z=2$: $2 = \frac{y}{x+y}$

$2x+2y=y$

$\Rightarrow 2x=-y$

$y=-2x$