1. Let $f(x, y) = 3x^2 + 6y^2$.

(a) [12 points] Find all local maxima, minima and saddle points for f. Does f have an absolute maximum and/or minimum? If so, where do they occur?

crit. pts: $f_x(\cancel{x},y) = 6x = 0 \Rightarrow x=0$
$f_y(x,y) = 12y = 0 \Rightarrow y=0$ $\Big\} \Rightarrow (0,0)$ only crit. pt.

$\left[\begin{array}{ll} f_{xx}(x,y) = 6 & f_{xy}(x,y) = 0 \\ f_{yy}(x,y) = 12 & \end{array}\right. \Rightarrow D(0,0) = (6)(12) - 0^2 > 0 \Rightarrow$ either max or min

$f_{xx}(0,0) = 6 > 0 \Rightarrow$ conc. up $\Rightarrow$ local min @ $(0,0)$.

Also have absolute min at $(0,0)$ because $f(0,0) = 0$ and $f(x,y) > 0$ if $(x,y) \neq (0,0)$. There is no absolute max value of f.

(b) [10 points] Use Lagrange multipliers to find any maxima and minima of f where now we add the constraint that (x, y) satisfies $x^2 + y^2 = 1$.

$g(x,y) = x^2+y^2$, $\vec{\nabla} f = \lambda \vec{\nabla} g \Rightarrow \langle 6x, 12y \rangle = \lambda \langle 2x, 2y \rangle$

$\Rightarrow \left[\begin{array}{l} 6x = \lambda 2x \\ 12y = \lambda 2y \\ x^2+y^2=1 \end{array}\right.$ $\rightarrow$ either: $\lambda = 3$ OR $x = 0$

$\lambda = 3 \Rightarrow 12y = 6y \Rightarrow y = 0 \Rightarrow 0^2 + x^2 = 1 \Rightarrow x = \pm 1$

$\Rightarrow 0^2 + y^2 = 1 \Rightarrow y = \pm 1$ and $\lambda = 6$.

extrema are at $(0, \pm 1)$ and $(\pm 1, 0)$.

Since $f(0, \pm 1) = 6 \leftarrow$ max value of f on $x^2+y^2=1$

$f(\pm 1, 0) = 3 \leftarrow$ min value of f on $x^2+y^2=1$

(c) [8 points] What are the absolute maximum and minimum values of f on the entire closed bounded domain $x^2 + y^2 \leq 1$? (If they exist)

Putting (a) + (b) together, we see that

abs max value of f on $x^2+y^2 \leq 1$ is 6, $f(0,\pm 1) = 6$.

and abs min value of f on $x^2+y^2 \leq 1$ is 0 b/c $f(0,0) = 0$.