5.

(a) [10 points] Use a triple integral to find the formula for the volume of a solid sphere of radius A.

$$\mathrm{Vol(Sphere)} = \iiint_{\text{Sphere}} 1\, dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{A} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{A^3}{3}\sin\phi \, d\phi\, d\theta = \frac{A^3}{3}\int_0^{2\pi} (-\cos\phi)\Big|_0^{\pi} d\theta$$

$$= \frac{A^3}{3}\int_0^{2\pi}(1+1)\, d\theta = \frac{2A^3}{3}\cdot 2\pi = \frac{4\pi A^3}{3}$$

(b) [10 points] Define the parametrization $\vec{r}$ by $\vec{r}(u,v) = \langle A\cos v\ , A\sin v\ , u\rangle$ - the cylinder of radius A. Use integration to find the formula for the surface area of the cylinder of radius A and height H. (This means $0 \le u \le H$ and $0 \le v \le 2\pi$.)

$$\text{Surface Area} = \int_0^{2\pi}\int_0^{H} |\vec{r}_u \times \vec{r}_v|\, du\, dv = \int_0^{2\pi}\int_0^{H} A\, du\, dv$$

$$\parallel$$

$$\iint_S 1\, dS \qquad = 2\pi A H.$$

$$\vec{r}_u = \langle 0, 0, 1\rangle$$

$$\vec{r}_v = \langle -A\sin v, A\cos v, 0\rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle -A\cos v, -A\sin v, 0\rangle$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{A^2\cos^2 v + A^2\sin^2 v} = A$$