1. The following parts will lead to approximating the integral

$$\int_0^4 \int_1^3 xy \, dx \, dy \qquad \Rightarrow \qquad 1 \le x \le 3$$
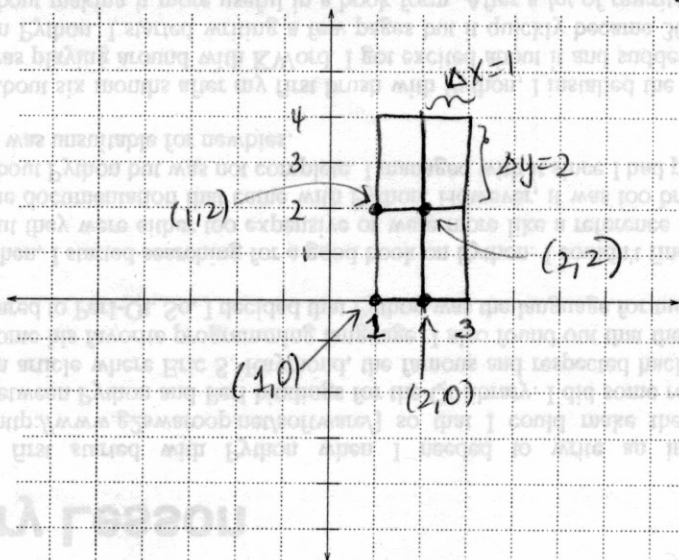$$0 \le y \le 4$$

by a double Riemann sum.

(a) [4 points] Let m = 2 and n = 2. What are Δx and Δy?

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{2} = 1$$

$$\Delta y = \frac{4-0}{2} = 2$$

(b) [6 points] Sketch the region of integration below and divide it into the appropriate number of subrectangles, each having dimensions Δx by Δy. For each subrectangle, find the coordinates of the lower left corner and lable them on your sketch.

(c) [10 points] Use the lower left corners to estimate the above double integral by a double Riemann sum.

$$\int_0^4 \int_1^3 xy \, dx \, dy \approx \sum_{i=0}^{2} \sum_{j=0}^{2} f(x_{ij}^*, y_{ij}^*) \Delta x \Delta y$$

$$f(x,y) = xy$$

$$= f(1,0)(1)(2) + f(2,0)(1)(2) + f(1,2)(1)(2) + f(2,2)(1)(2)$$

$$= 0 + 0 + 4 + 8 = 12$$