1. [3 point] Give the order, state which is the independent/dependent variable, and why the differential equation is or is not linear:

$$8\frac{d^4y}{dx^4} = x(1-x)\ .$$

order - 4th
independent variable - x
dependent variable - y

This equation is linear because the dependent variable appears only in the term $8\frac{d^4y}{dx^4}$, and no others. It fits the form of a linear eq'n:
$a_4(x)\frac{d^4y}{dx^4} + a_3(x)\frac{d^3y}{dx^3} + a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$
$\|$
$x(1-x)$

2. [1 point] Write a differential equation that fits the physical description: The rate of change of the population p of bacteria at time t is proportional to the population at time t.

$\frac{dp}{dt} = kp$ where k is the proportionality constant.

3. [3 points] Determine whether $x(t) = \cos(t)$ is a solution to the differential equation $\frac{dx}{dt} + tx = \sin(2t)$. Show why or why not.

$x(t) = \cos(t)$
$\Rightarrow x'(t) = -\sin(t)$

So $\frac{dx}{dt} + tx = \sin(2t)$

becomes $-\sin(t) + t\cos(t) = \sin(2t)$.

This is equality ← is not true over any interval

So the function is not a solution.

4. [3 points] Determine whether $e^{xy} + y = x - 1$ is an implicit solution to $\frac{dy}{dx} = \frac{1-e^{xy}y}{1+e^{xy}x}$. Show why or why not.

implicit differentiation yields: $\frac{d}{dx}\left(e^{xy} + y\right) = \frac{d}{dx}(x-1)$

chain rule: $\Rightarrow e^{xy}\frac{d}{dx}(xy) + \frac{dy}{dx} = 1$

product rule: $\Rightarrow e^{xy}\left[y + x\frac{dy}{dx}\right] + \frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx}\left(xe^{xy} + 1\right) = 1 - ye^{xy}$

$\Rightarrow \frac{dy}{dx} = \frac{1-ye^{xy}}{1+xe^{xy}}$

↑ Same as original o.d.e. so it is a solution.