1. Solve the o.d.e.

$$(x^2+1)\frac{dy}{dx} + xy - x = 0 \; .$$

Linear: $\frac{dy}{dx} + \frac{x}{x^2+1} y = \frac{x}{x^2+1}$

$u = x^2+1$
$du = 2x\,dx$

$\mu = e^{\int \frac{x}{x^2+1} dx} = e^{\int \frac{1}{2}\frac{1}{u} du} = e^{\ln|u|^{1/2}} = \sqrt{x^2+1}$

$\Rightarrow (x^2+1)^{1/2} \frac{dy}{dx} + x(x^2+1)^{-1/2} y = x(x^2+1)^{-1/2}$

$\frac{d}{dx}\left((x^2+1)^{1/2} y \right) = x(x^2+1)^{-1/2}$

$\Rightarrow (x^2+1)^{1/2} y = \int \frac{1}{2} u^{-1/2} du = u^{1/2} + C$

$\boxed{y = 1 + C(x^2+1)^{-1/2}}$

2. Solve the intial value problem

$$\left(\frac{1}{x} + 2y^2x\right) dx + \left(2yx^2 - \cos(y)\right) dy = 0$$

with $y(1) = \pi$.

Exact? $\frac{\partial M}{\partial y} = 4yx = \frac{\partial N}{\partial x}$ yes!

$\Rightarrow \frac{\partial f}{\partial x} = \frac{1}{x} + 2y^2x$ and $\frac{\partial f}{\partial y} = 2yx^2 - \cos(y)$

$\Rightarrow f(x,y) = \int \frac{1}{x} + 2y^2x \, dx = \ln|x| + y^2x^2 + g(y)$

$\Rightarrow \frac{\partial f}{\partial y} = 2yx^2 + g'(y) = 2yx^2 - \cos(y)$

$\Rightarrow g'(y) = -\cos(y) \Rightarrow g(y) = -\sin(y).$

So general sol'n is $\boxed{\ln|x| + y^2x^2 - \sin(y) = C}$

and $y(1) = \pi \Rightarrow \ln|1| + \pi^2 - \sin(\pi) = C \Rightarrow \boxed{\pi^2 = C}$

So sol'n to IVP is $\boxed{\ln|x| + y^2x^2 - \sin(y) = \pi^2}$