1. Solve the initial value problem

$$\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x ,$$

with $y(1) = 1$.

$$\frac{dy}{dx} + \frac{3}{x} y = 3x - 2 \quad \leftarrow \text{standard linear form w/ } P(x) = 3/x.$$

$$\text{let } \mu(x) = e^{\int \frac{3}{x} dx} = e^{3\ln|x|} = x^3$$

sol'n:

$$\boxed{y = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{9}{10x^3}}$$

$$\Rightarrow \quad \frac{d}{dx}(x^3 y) = 3x^4 - 2x^3$$

$$\Rightarrow \quad x^3 y = \frac{3}{5}x^5 - \frac{1}{2}x^4 + C \quad \Rightarrow y = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{C}{x^3}$$

$$y(1) = \frac{3}{5} - \frac{1}{2} + C = 1 \quad \Rightarrow C = 1 + \frac{1}{2} - \frac{3}{5} = \frac{3}{2} - \frac{3}{5} = \frac{15}{10} - \frac{6}{10} = \frac{9}{10}.$$

2. Determine whether or not the equation is exact. If it is, then solve it.

$$\left(\frac{t}{y}\right) dy + (1 + \ln(y))dt = 0 .$$

$$\frac{\partial M}{\partial t} = \frac{1}{y}, \quad \frac{\partial N}{\partial y} = \frac{1}{y} \Rightarrow \text{Yes, it is exact!}$$

$$\Rightarrow \quad \frac{\partial f}{\partial y} = \frac{t}{y} \quad \& \quad \frac{\partial f}{\partial t} = 1 + \ln(y)$$

$$\Rightarrow \quad f(t,y) = \int \frac{t}{y} dy = t\ln|y| + g(t)$$

$$\Rightarrow \quad \frac{\partial f}{\partial t} = \ln|y| + g'(t) = 1 + \ln|y| \quad \Rightarrow g'(t) = 1$$

$$\Rightarrow g(t) = t.$$

Gen. sol'n: $\boxed{t\ln|y| + t = C}$