Department of Mathematics
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Math 231:Introduction to Ordinary Differential Equations
Mini-Project: Left/Right Curling Snails (Evolution)

Consider two types of animals, say left-curling snails and right-curling snails, that compete for the same resources to survive and reproduce. Let L denote the number of left-curling snails (in millions) and R the number of right-curling snails (in millions). These two populations compete for the same resources. We might propse the following equations to govern the dynamics of the populations:

$$\frac{dR}{dt} = g_1 R(1 - (R + L)) - a_1 RL \tag{1}$$

$$\frac{dL}{dt} = g_2 L(1 - (R + L)) - a_2 LR \tag{2}$$

where the a_i's are constant, crude measures of the extent to which right and left curling snails interaction affects the growth of each population, respectively, and the g_i's are a measure of the growth rate of each snail type at low total snail population density, respectively.

The article "Left Snails and Right Minds" appear in Natural History, in April of 1995 and was written by Stephen J. Gould. He went on at length with examples in the natural world of the dominance of the right-handed shell, and inquired as to why it is so. The question here is whether or not this simple competition model can predict the observed phenomenon of near absence of the left-curling shells over a long time period?

My intention for a group of three is that you do 1 and 5 as a group, and each member is to take responsibility for one of 2,3, or 4.

1. Explain the model - describe what each term on the right hand sides represents and why it is included. What would it mean for $a_1 < a_2$? Or for $a_2 < a_1$? Or for $a_2 = a_1$? What would $g_1 < g_2$ mean? Or $g_1 > g_2$, or $g_1 = g_2$?

2. For this part we'll investigate the role of a_i's in the dynamics of snail evolution. First, let $g_1 = g_2 = 1$ and choose some a_1 and a_2 value so that $a_1 = a_2 > 0$ and do a full

phase plane analysis. List all equillibria. What are the possible long-term outcomes? And what do those possible long-term outcomes and equillibria represent happening in the snail populations?

Next, choose 3 different sets of initial conditions, each from a different region of the phase plane if possible, and use MATLAB to solve for $L(t)$ and $R(t)$ over time in each case. From the point of view of evolution, it is interesting to start by assuming that at our initial time, long long ago, there were nearly equal populations of left and right-curling snails, say $(L, R) = (.5, .51)$. Include at least one initial condition that is like this.

Next take values so that $a_1 = a_2 < 0$ and repeat. What would this represent in terms of the left/right snail interactions? What long term outcomes are possible here?

Finally, repeat the investigation for $a_1 = a_2 = 0$? What does this mean for the snail interactions? What long term outcomes are possible here?

Looking at your results for all three cases, and given that right curling snails are far more prevalent today than left curling snails, which of the three scenarios are possibly representative of the actual snail dynamics?

3. Now let's allow a_1 to be different than a_2. First choose values for each so that $a_1 > 0$ and $a_2 < 0$. What does this choice represent in terms of the interaction between the two types of snails? Investigate the dynamics now using a full phase plane, and use MATLAB to plot $L(t)$ and $R(t)$ for three different initial conditions that preferably come from three different regions of the phase plane.

   From the point of view of evolution, it is interesting to start by assuming that at our initial time, long long ago, there were nearly equal populations of left and right-curling snails, say $(L, R) = (.5, .51)$. Include at least one initial condition that is like this.

   Next, choose $a_1 < 0$ and $a_2 > 0$. Repeat the investigation above.

4. In this part, we'll investigate the effects of the g_i's on the snail dynamics. First, let $a_1 = a_2 = 1$, but let $g_1 = 1$ and $g_2 = 0.5$. What does this difference in g_i values represent? Construct the phase plane for this system, and use MATLAB to plot $L(t)$ and $R(t)$ for three different initial conditions that preferably come from three different regions of the phase plane.

   What are the possible long-term outcomes? What differences do you note between the dynamics here and what you saw in part 1? It would be interesting to take the same initial conditions for this part as you did in part 1 for your MATLAB experiments so that you can make a direct comparison between what happens over time for the two different models if they start off at the same place. Is this a likely model for reality?

Repeat the above for $g_1 = 0.5$ and $g_2 = 1$.

5. Summarize your findings. How does the threshold value of 0 for a_1 and a_2 effect the outcomes? How does the relative size (which one, if either, is greater) of g_1 and g_2 effect the outcomes?

   According to your work, which of these cases seems to represent the real life situation at hand? What does this tell us in terms of the "sociology" of the snails? Can you imagine any way in which you might experimentally test this hypothesis?